COV886 Special Module in Algorithms: Computational Social Choice

Lecture 3

Computational Barriers to Manipulation

Jan 29, 2022 | Rohit Vaish

Reminder about starting recording

Last Time

[Gibbard'73; Satterthwaite'75]

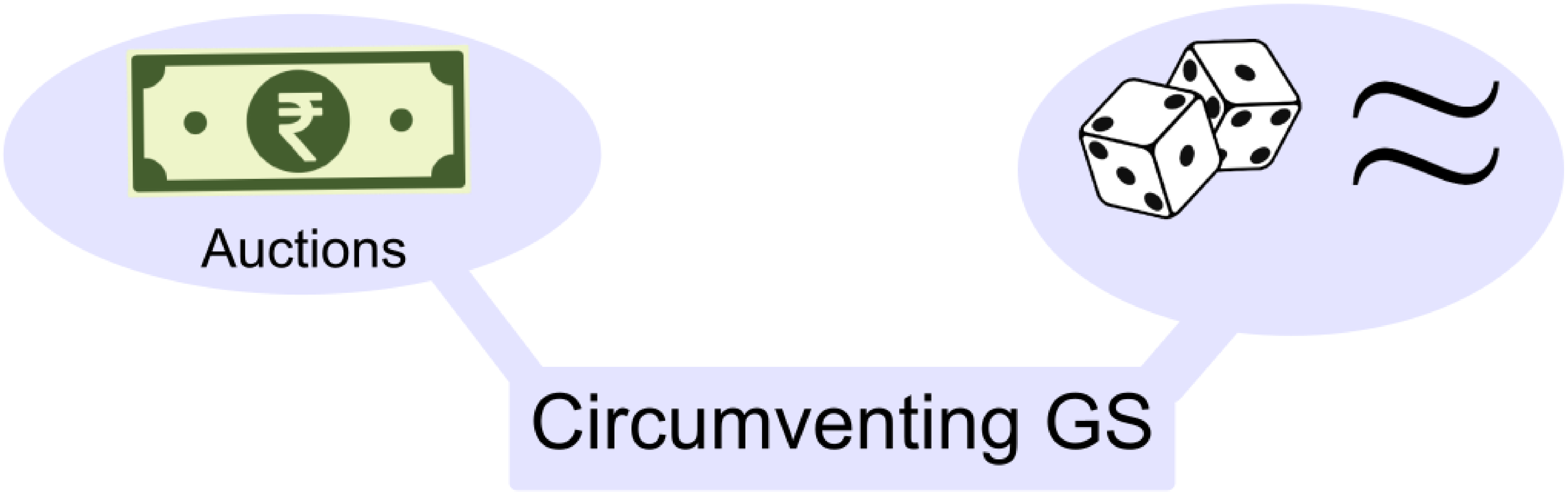
Any onto and non-dictatorial voting rule must be manipulable.

Circumventing GS

Circumventing GS

Auctions

Circumventing GS

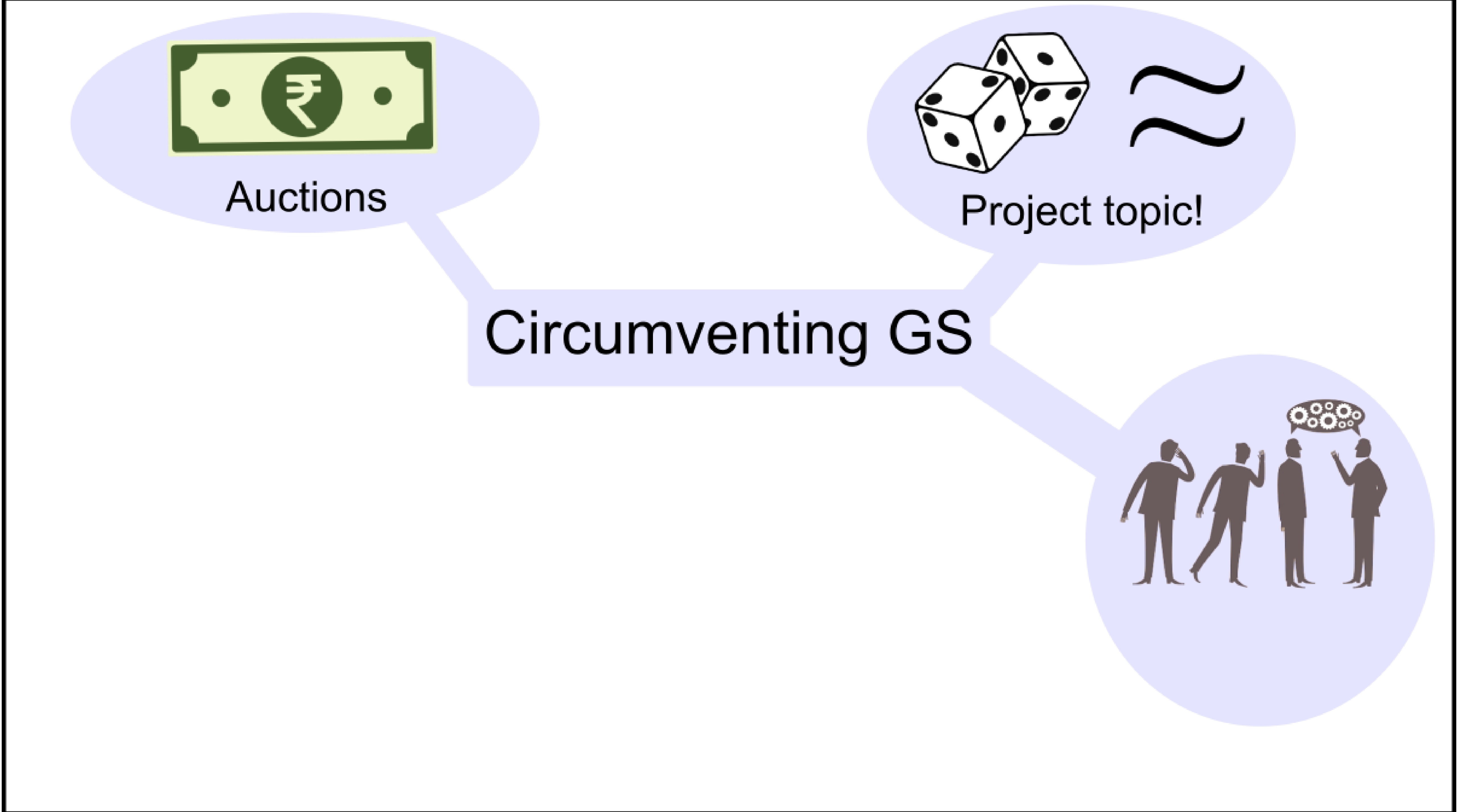

Auctions
Circumventing GS

Auctions

 $\approx$

Project topic!

Circumventing GS


Auctions
Project topic!
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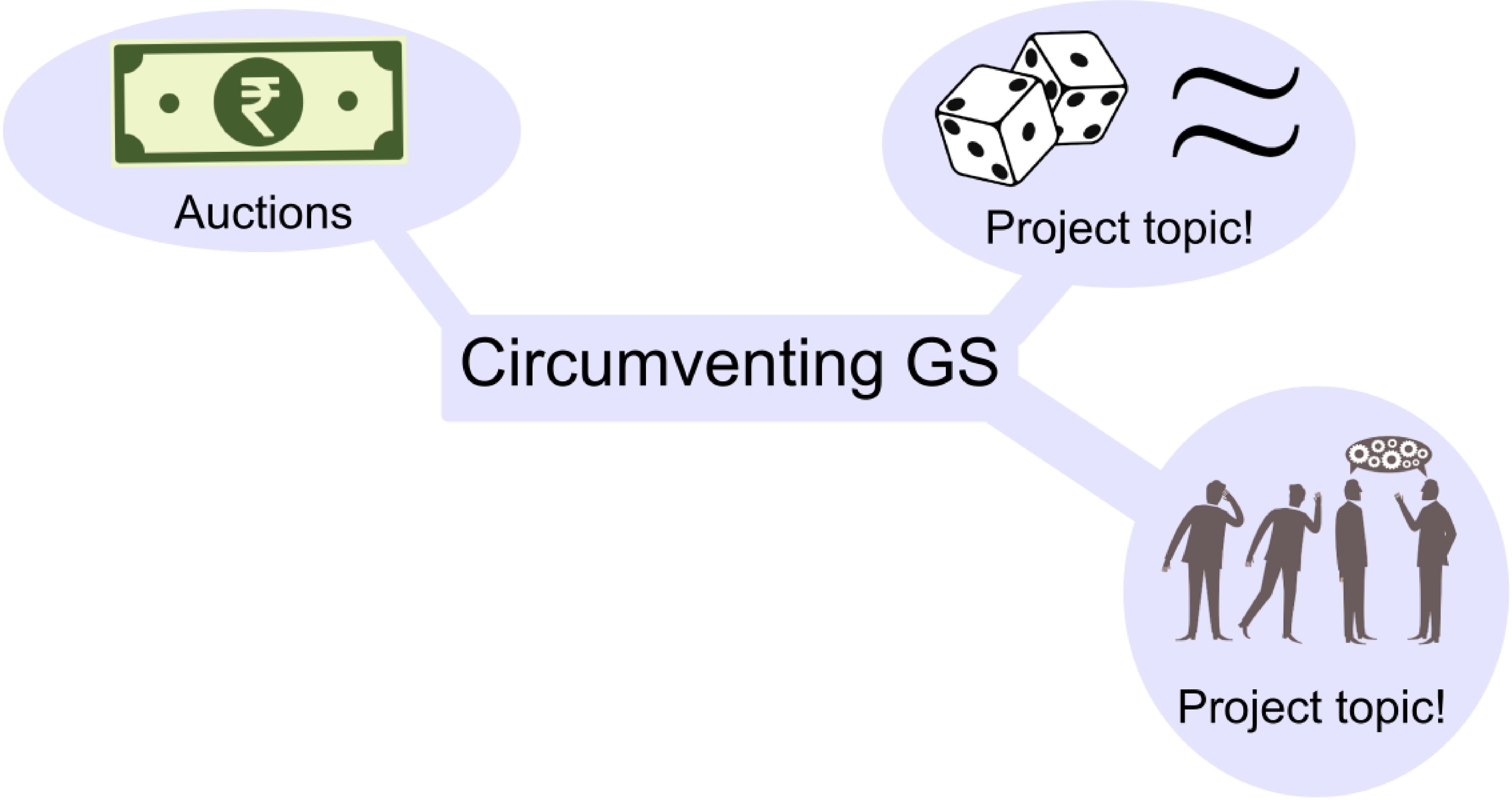
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Project topic!
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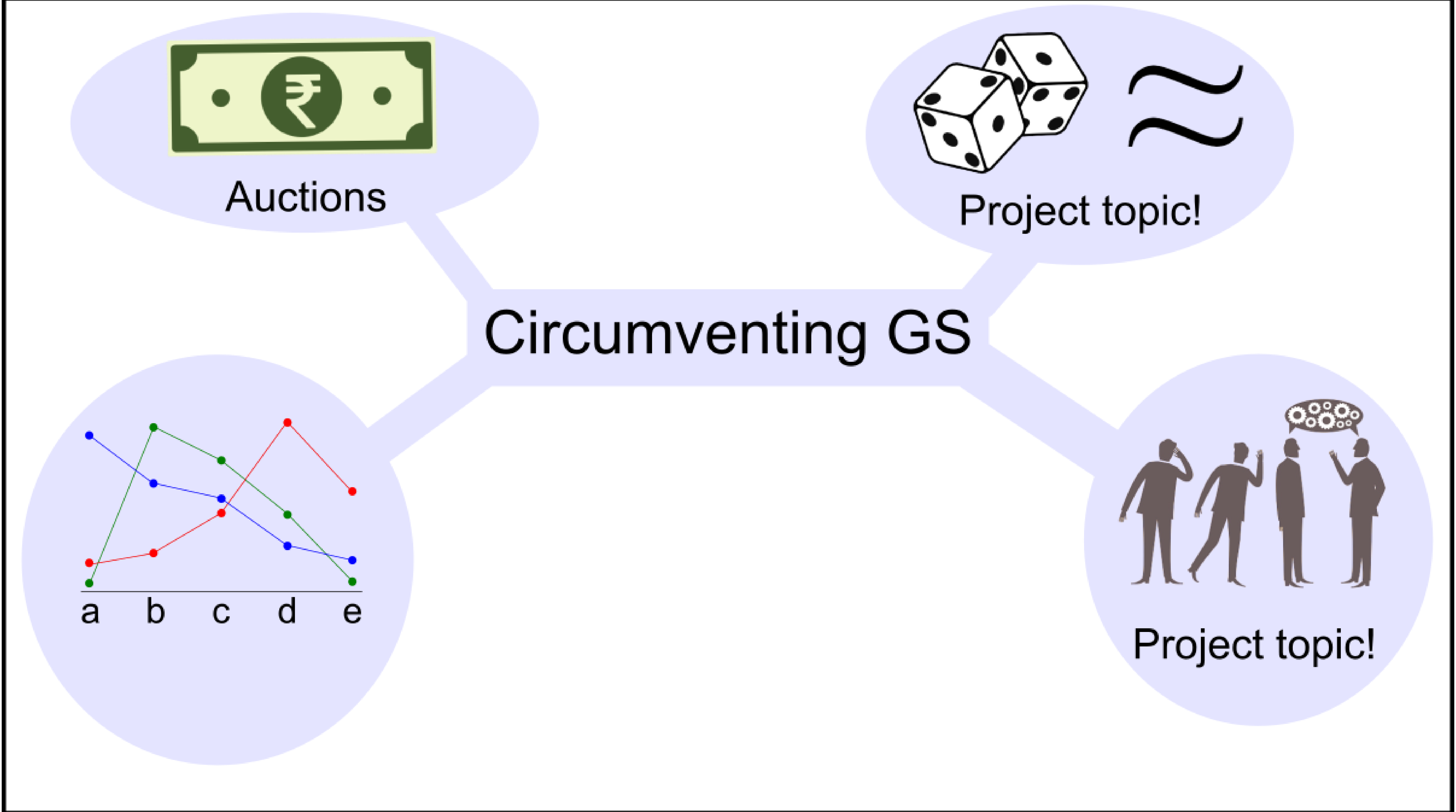
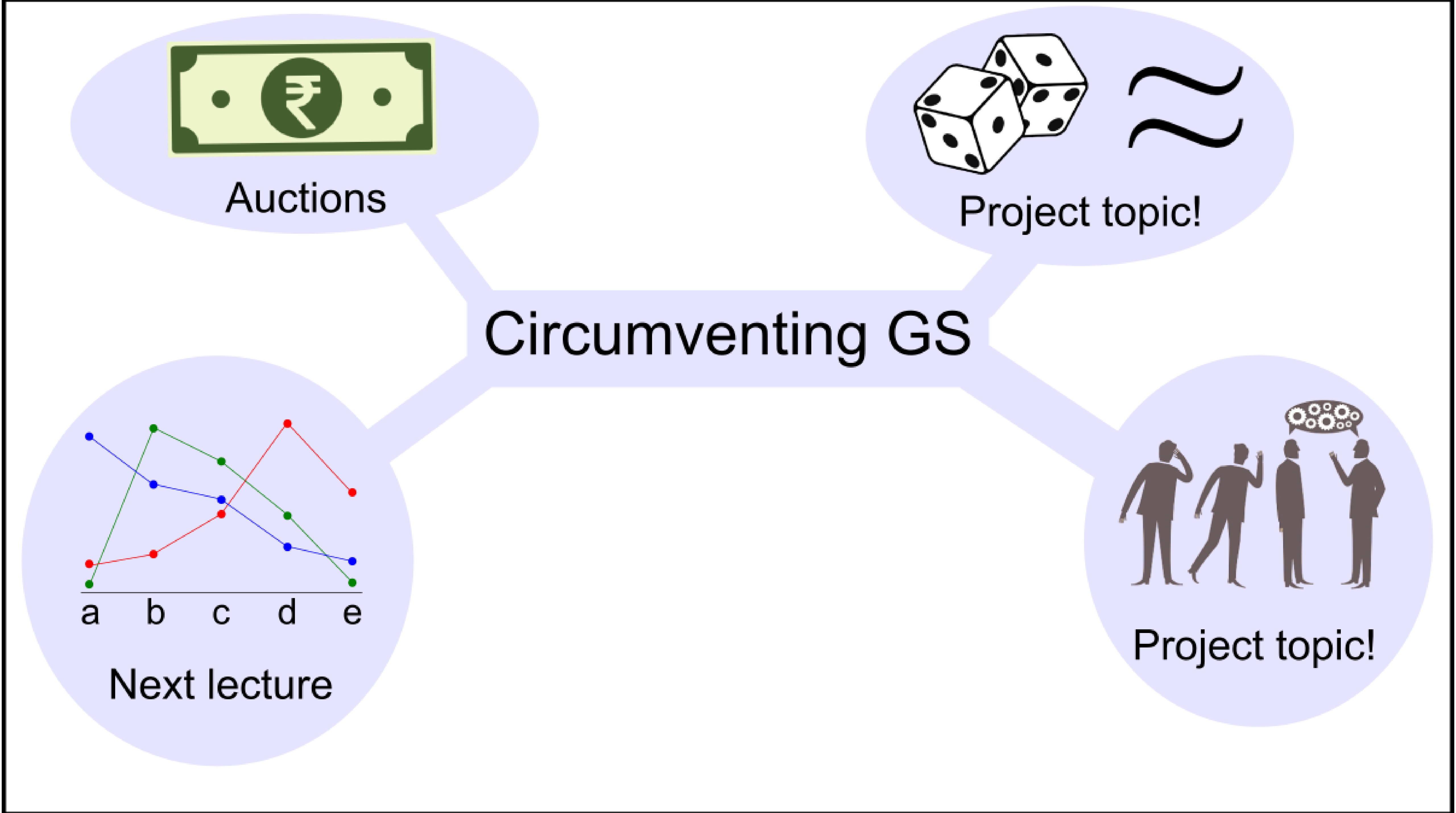


Auctions
Project topic!
Circumventing GS
a
b
c
d
e
Project topic!


Auctions
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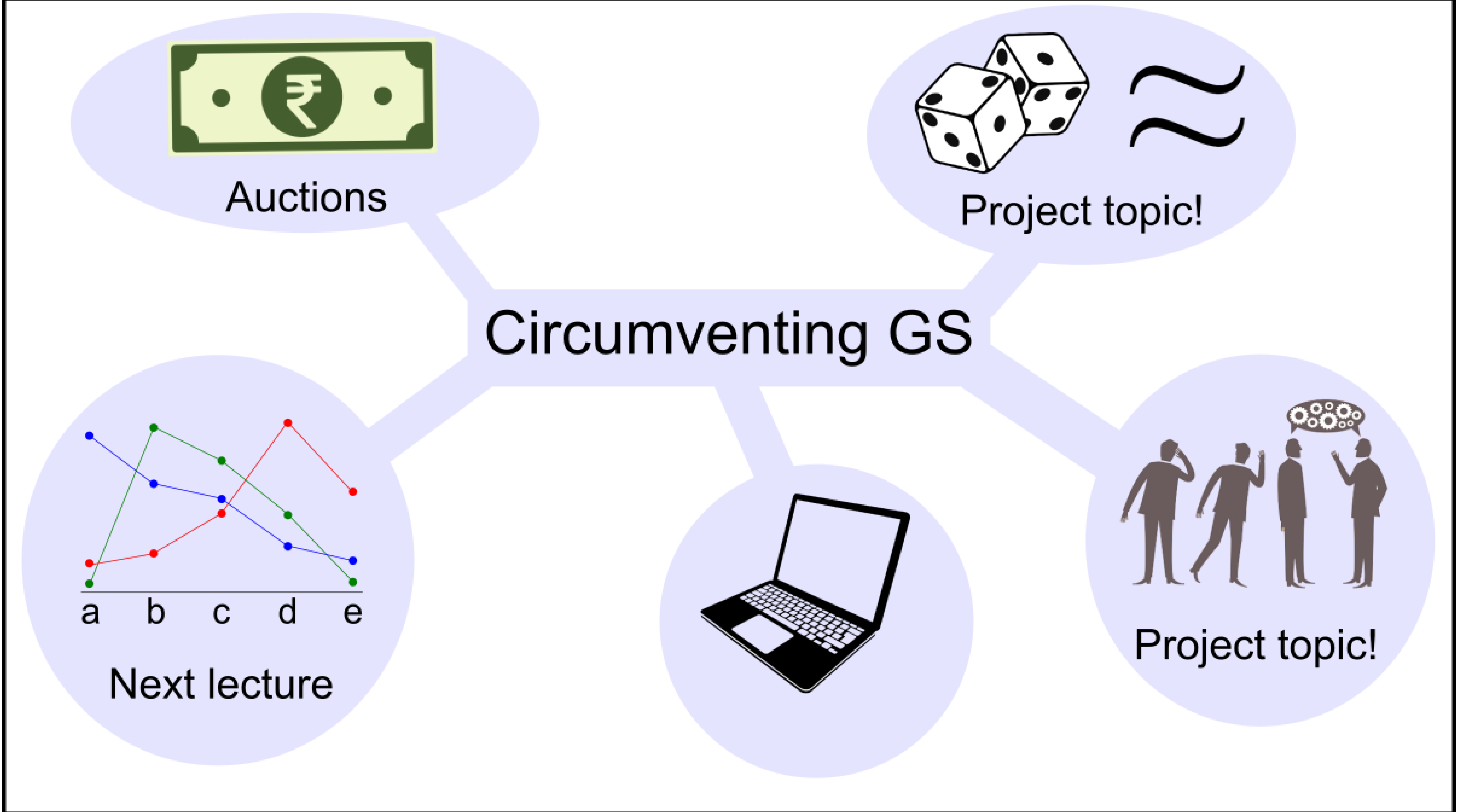
Next lecture
Project topic!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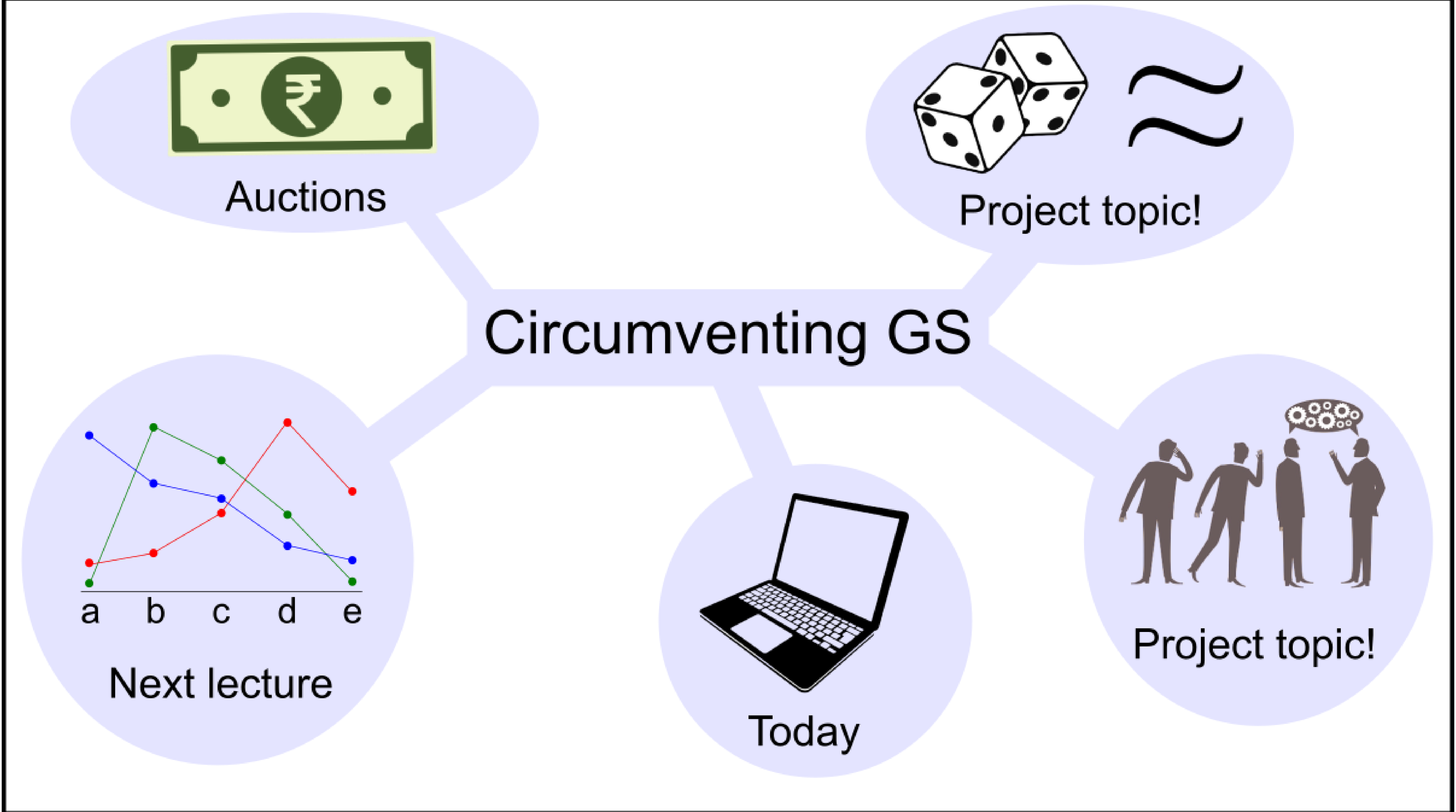

Auctions
Project topic!
Circumventing GS
a
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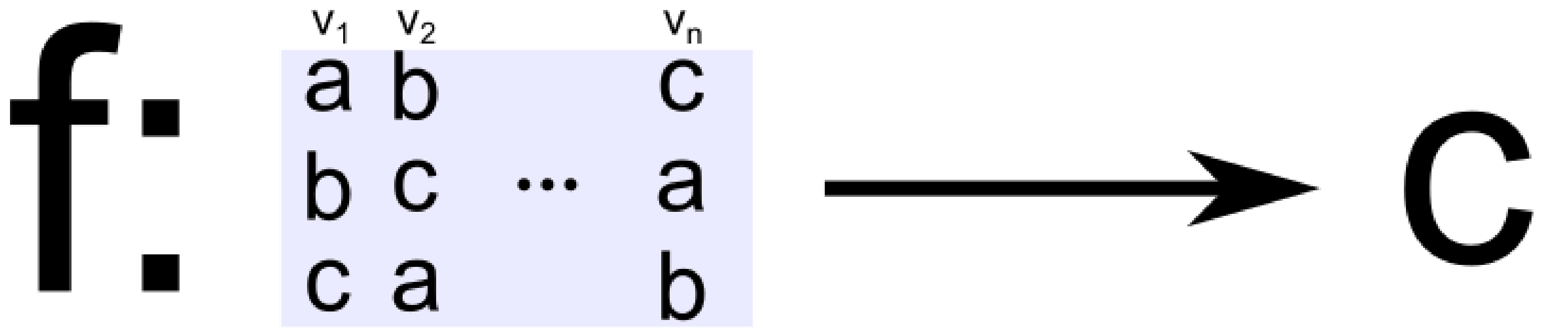
Next lecture
Today
Project topic!

VOTING RULE

A mapping from preference profiles to candidates.



f-Manipulation

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Input:

- A set of candidates and a set of voters $v_1, v_2, \ldots, v_n$

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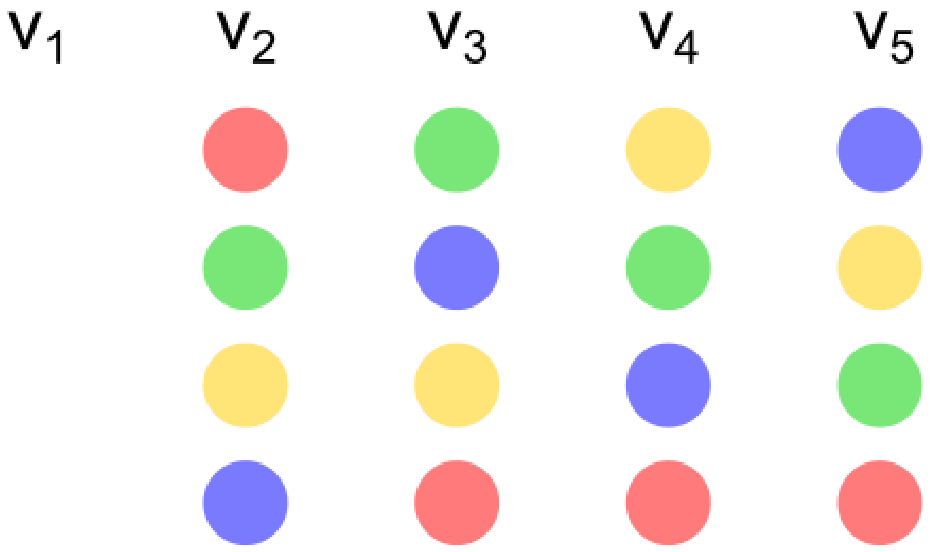
- Manipulator v_1's favorite candidate c

Question:

Does there exist a vote P_1 of the manipulator v_1 such that $f(P_1, P_2, \ldots, P_n) = c$?

Manipulation under Borda Count



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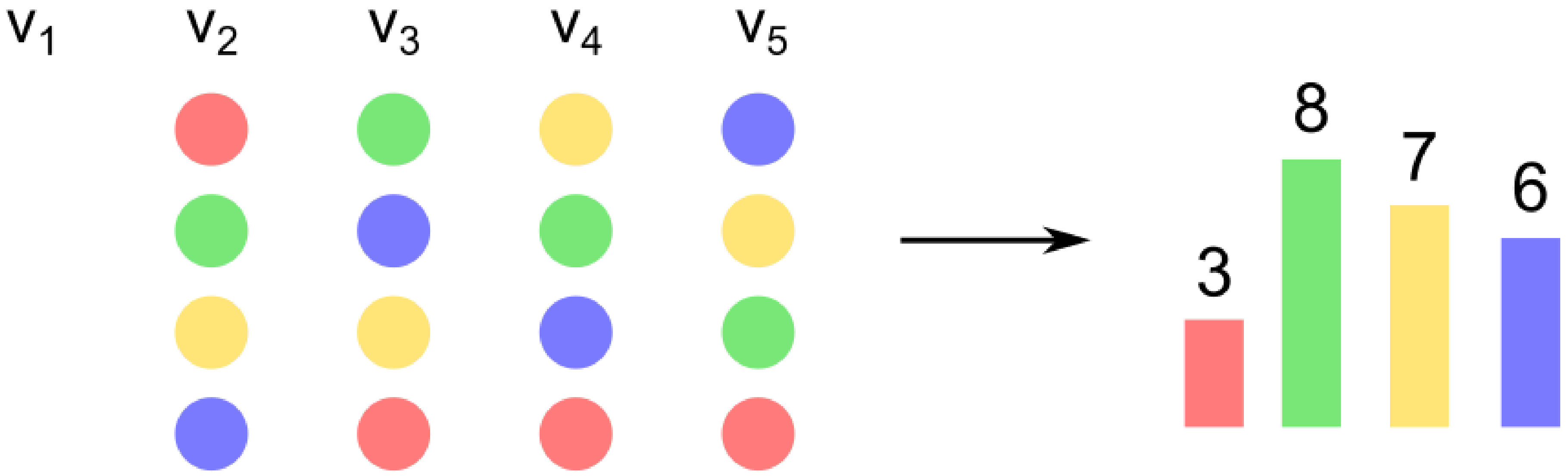

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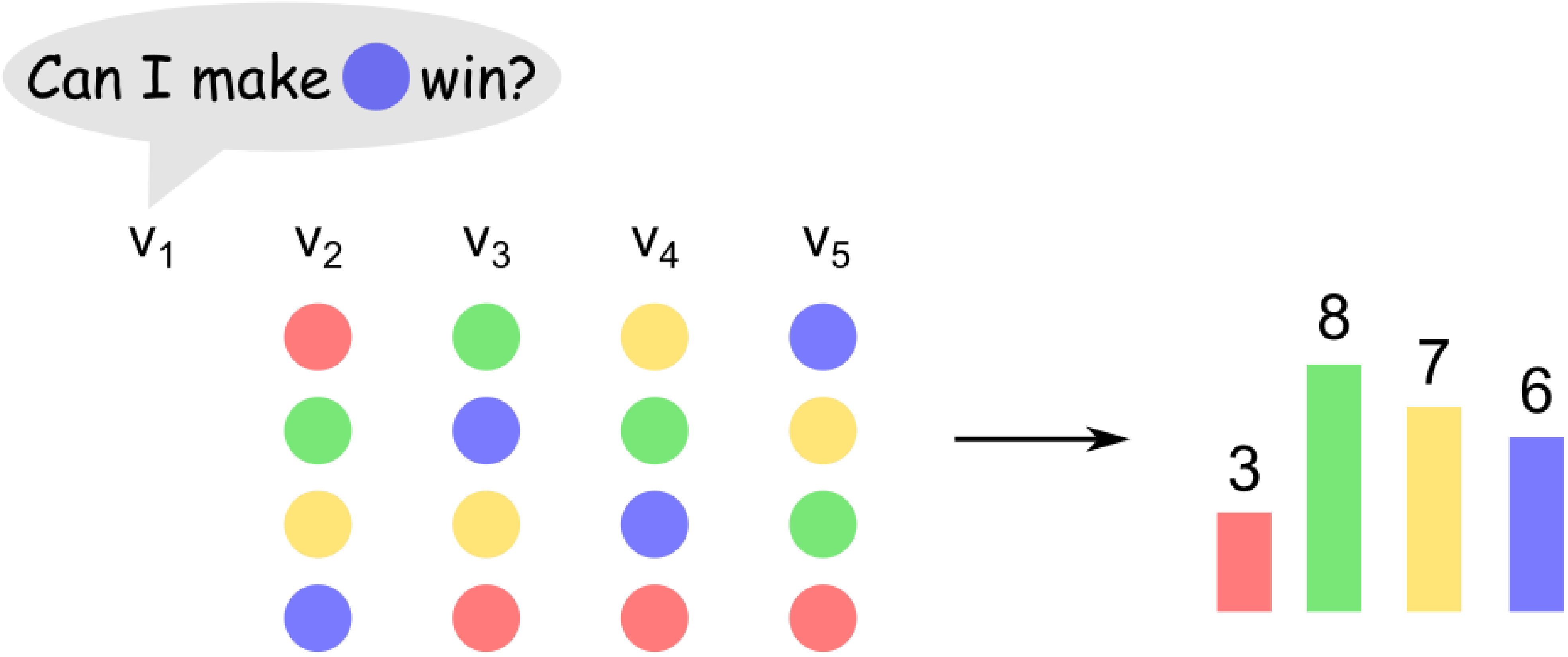

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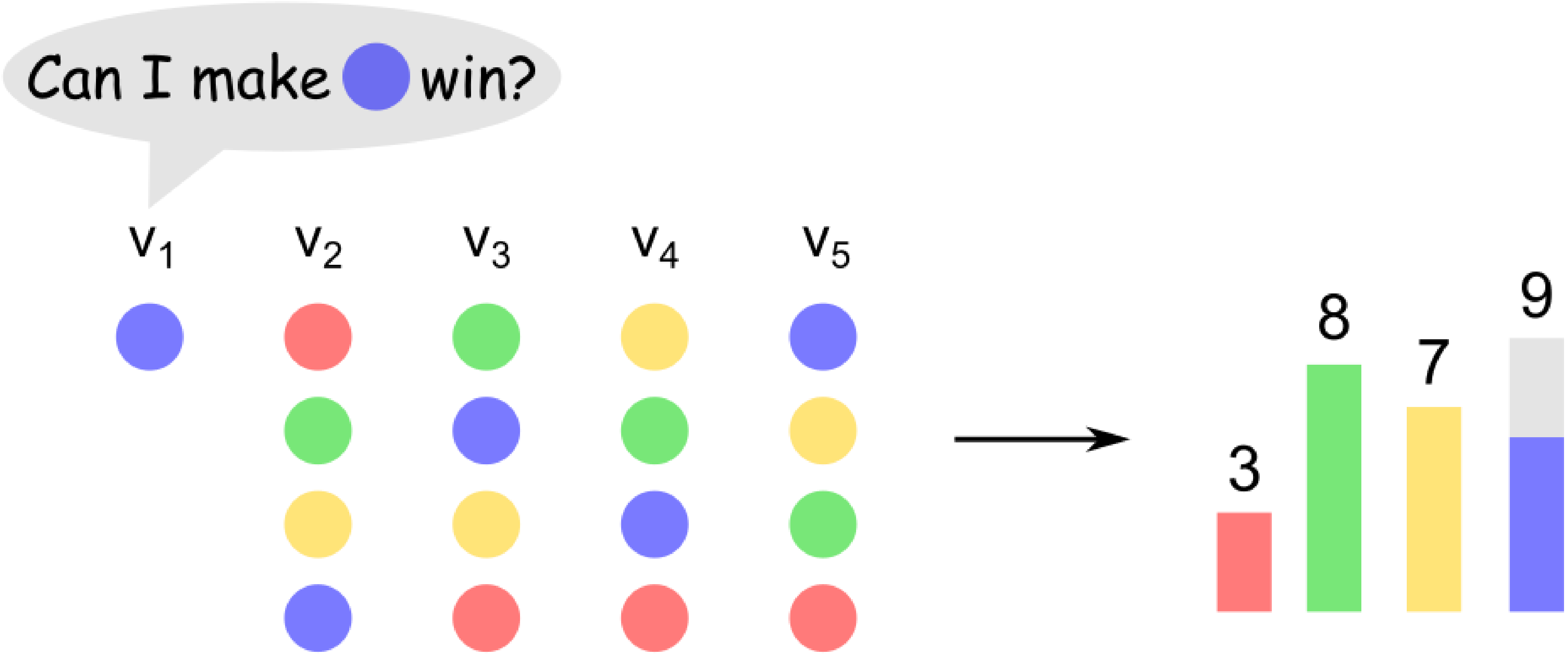

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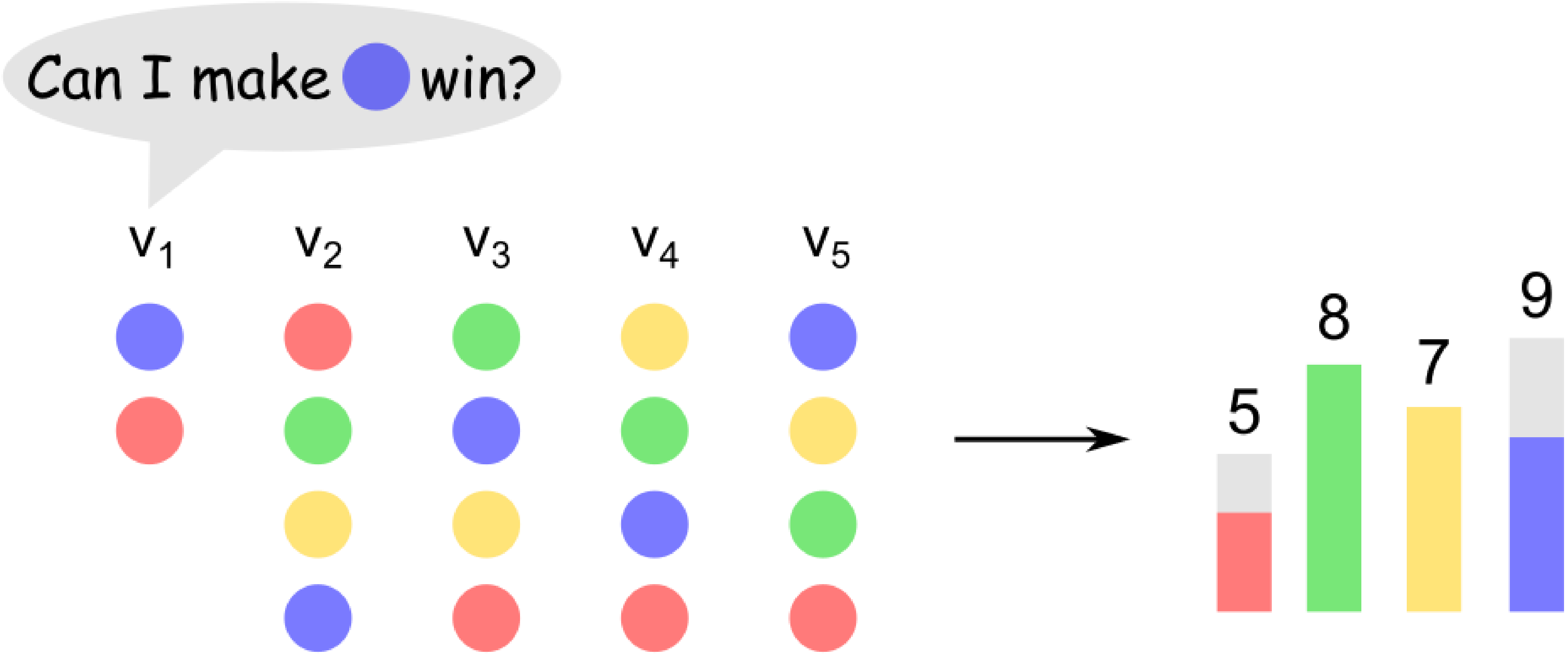

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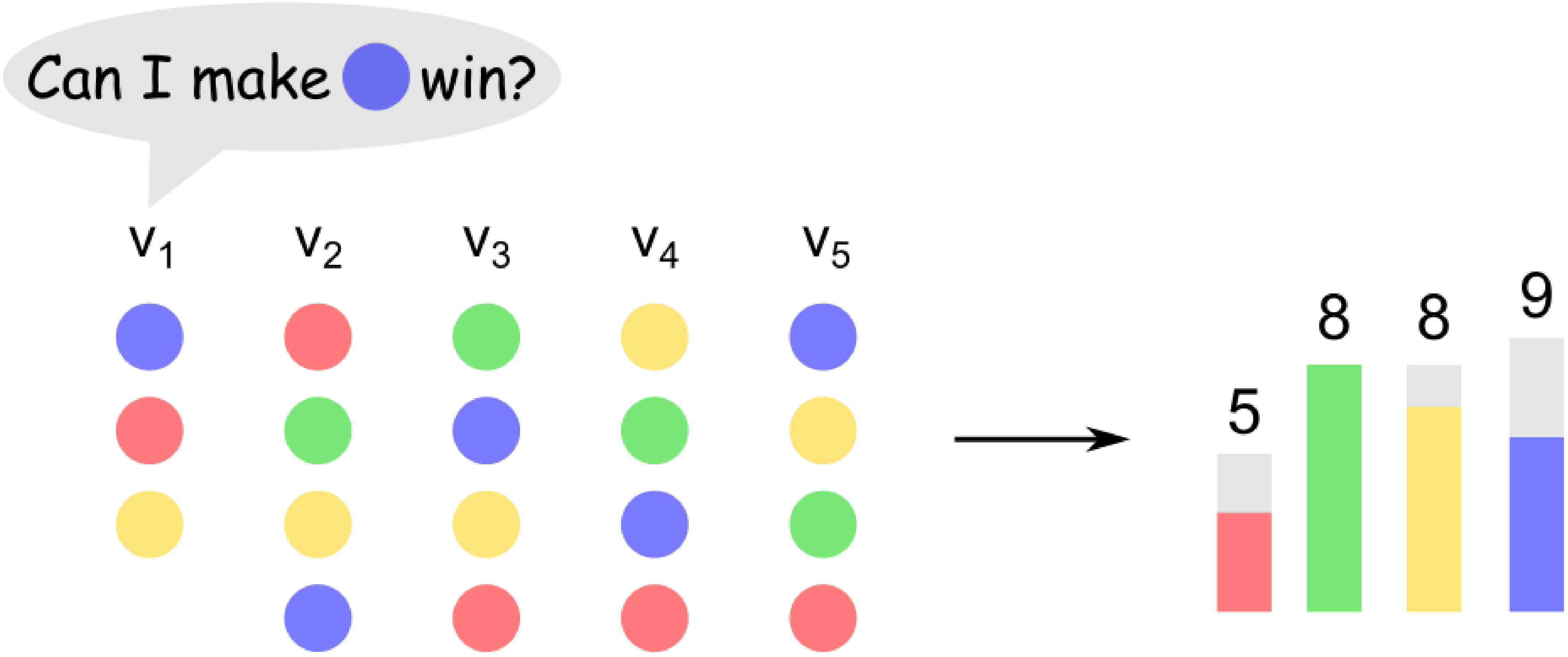

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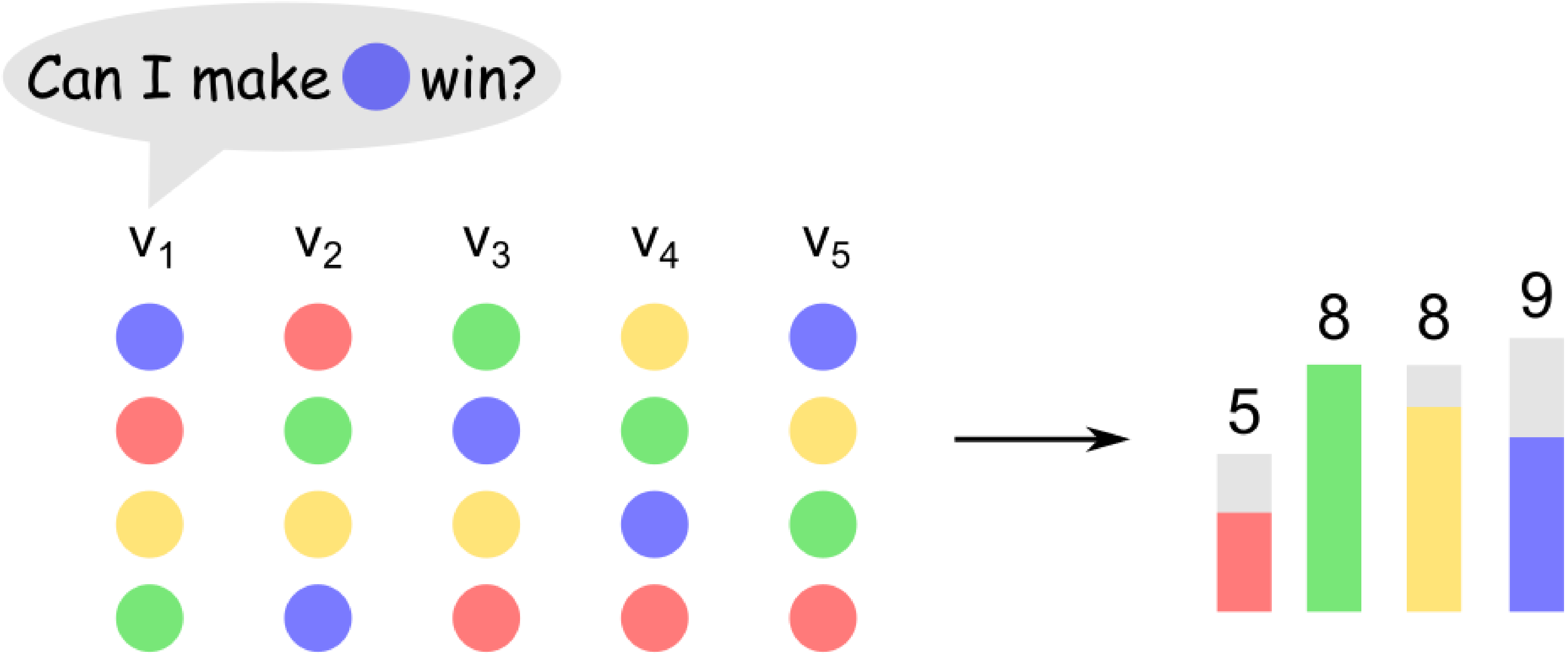

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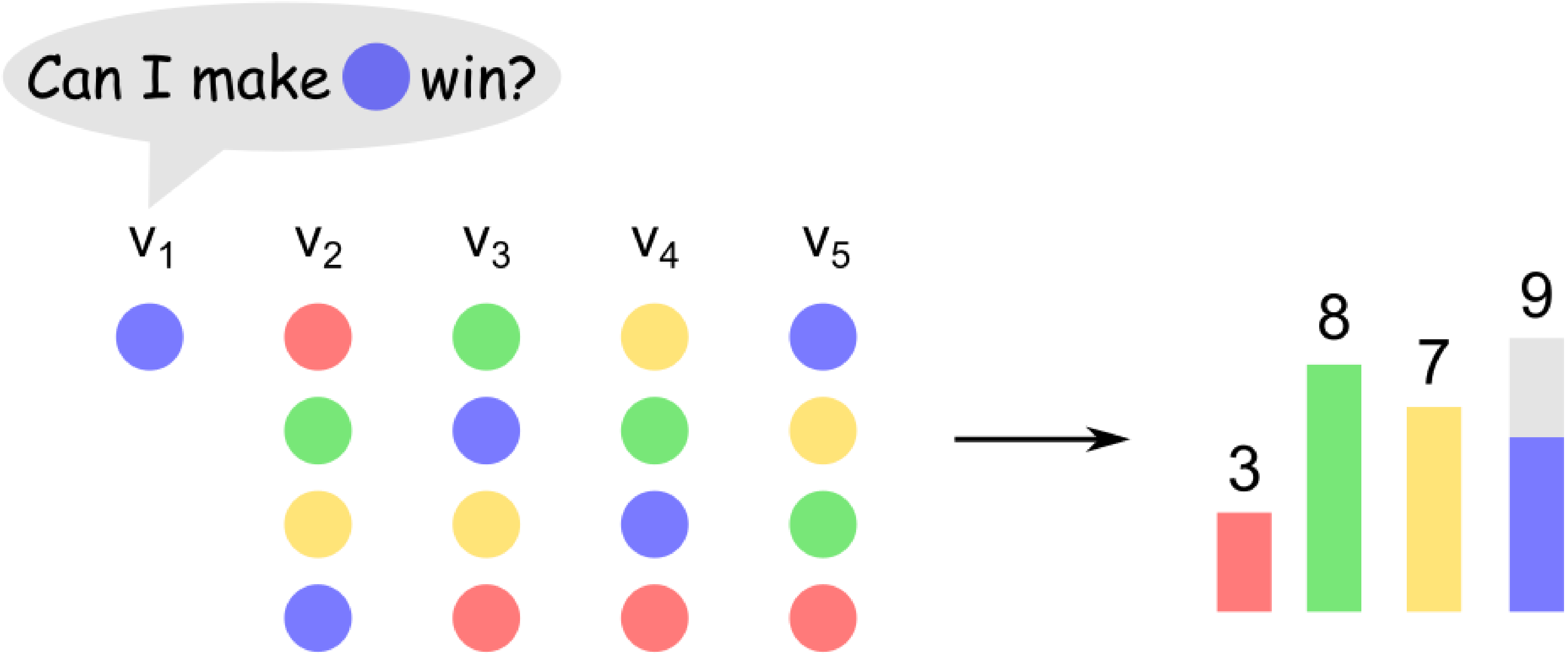
- Otherwise, return 'No'.

Manipulation under Borda Count



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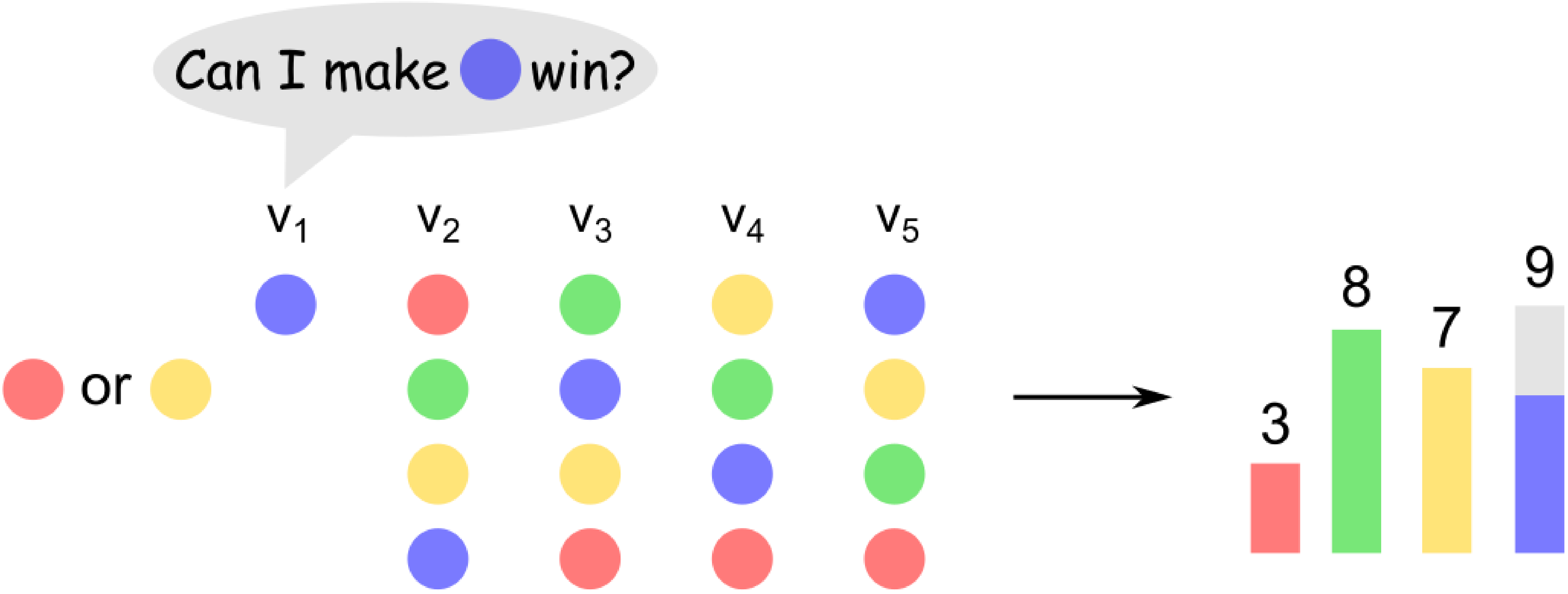

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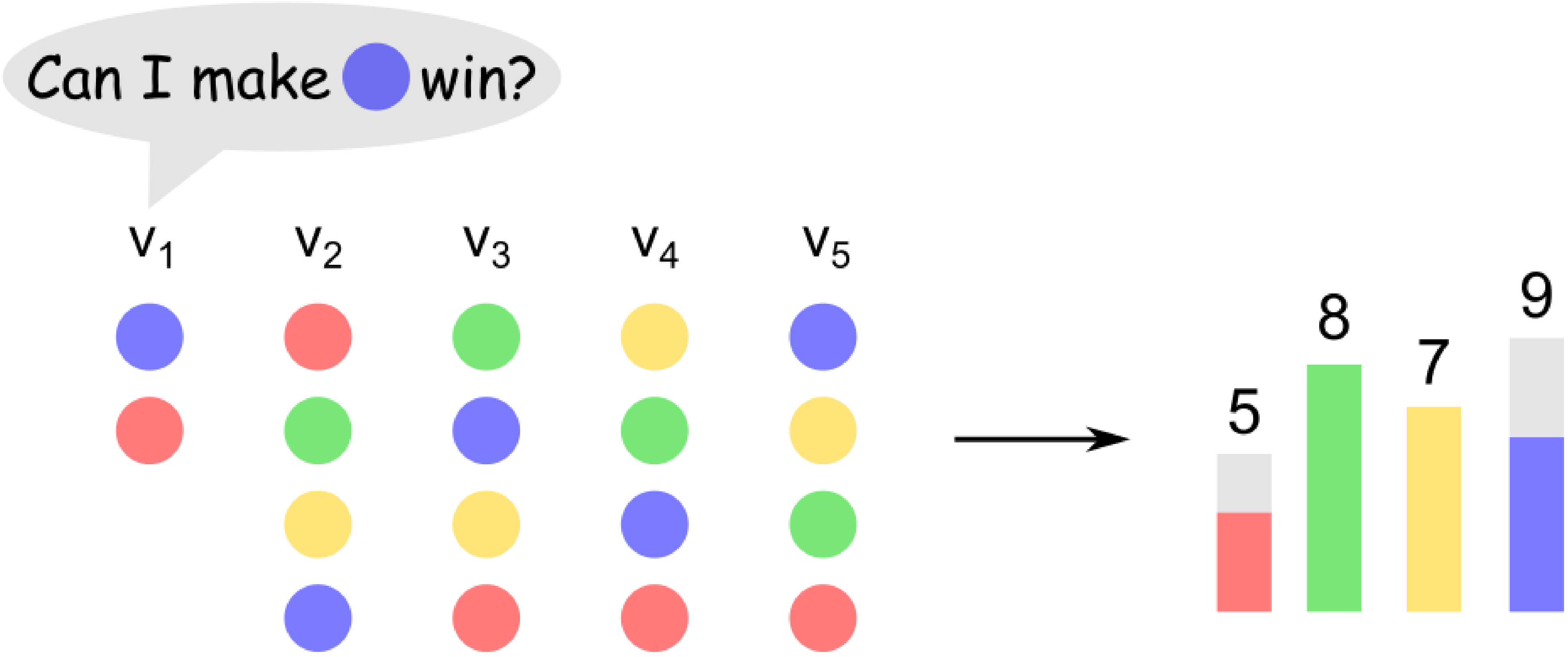

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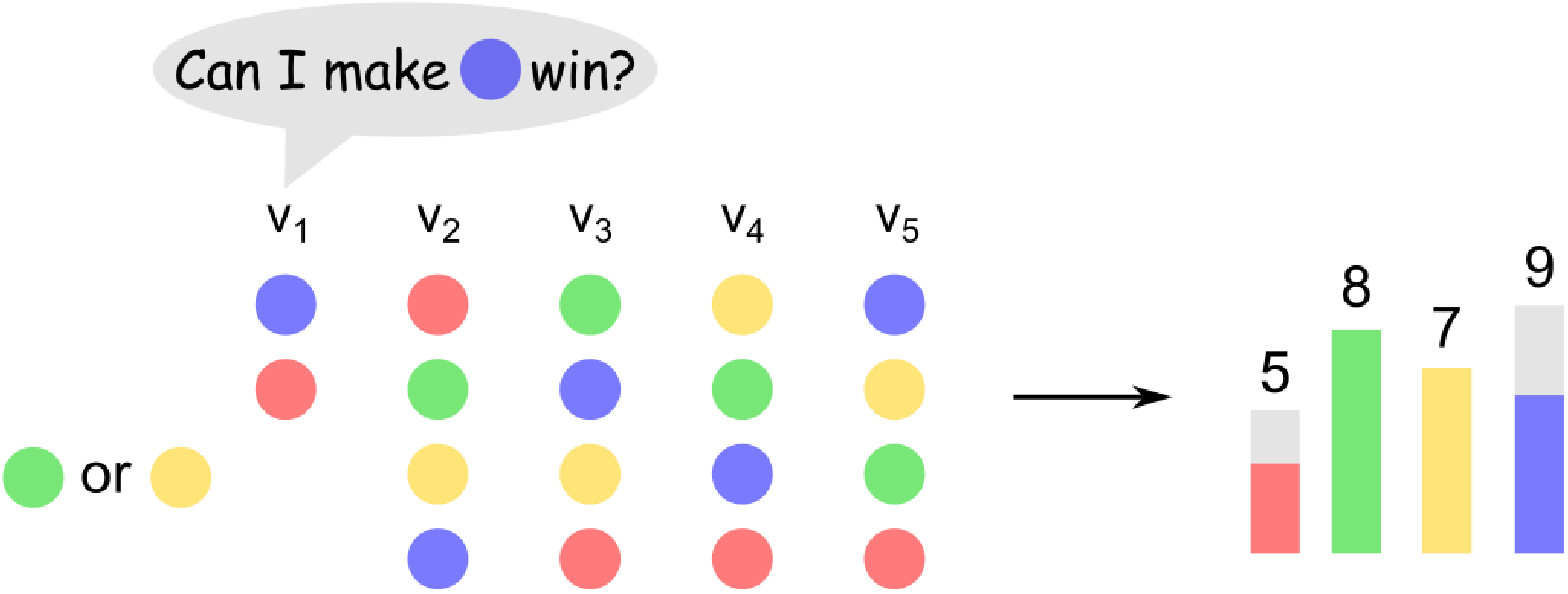

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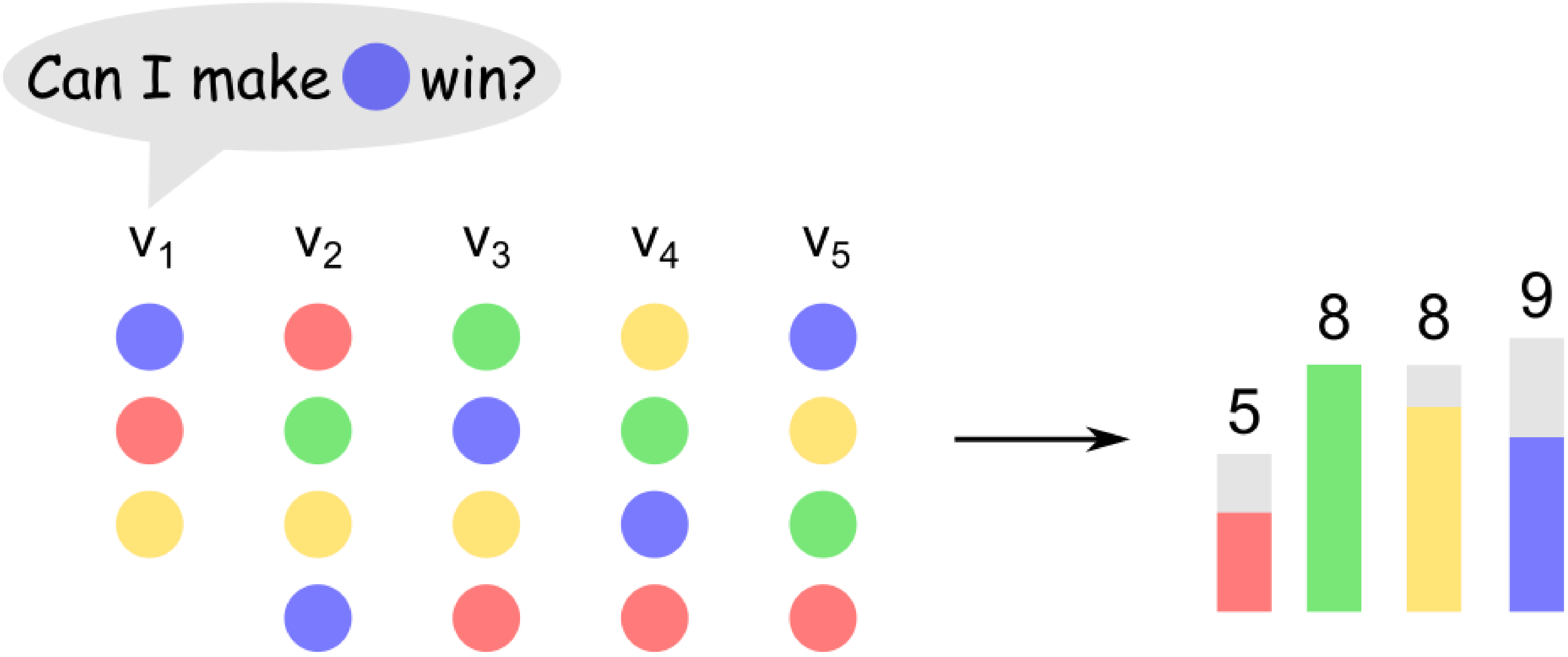

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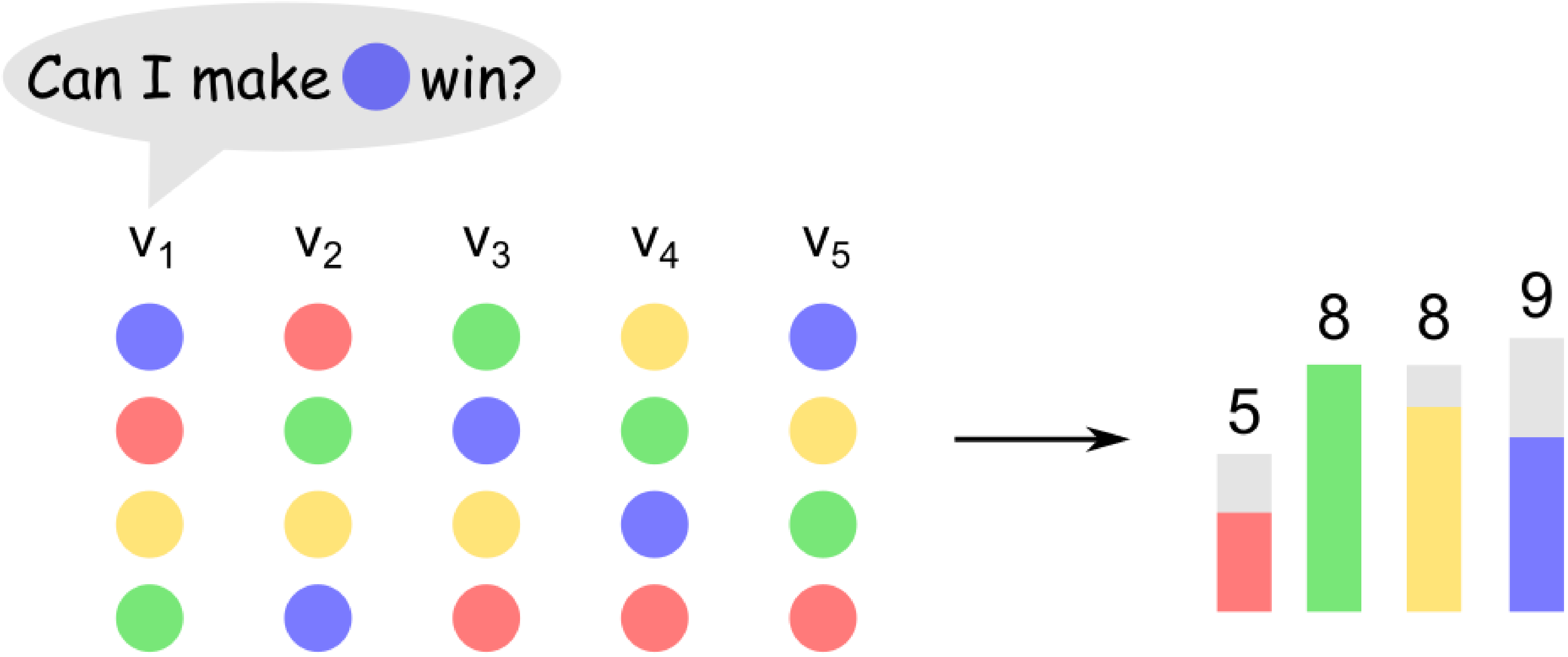
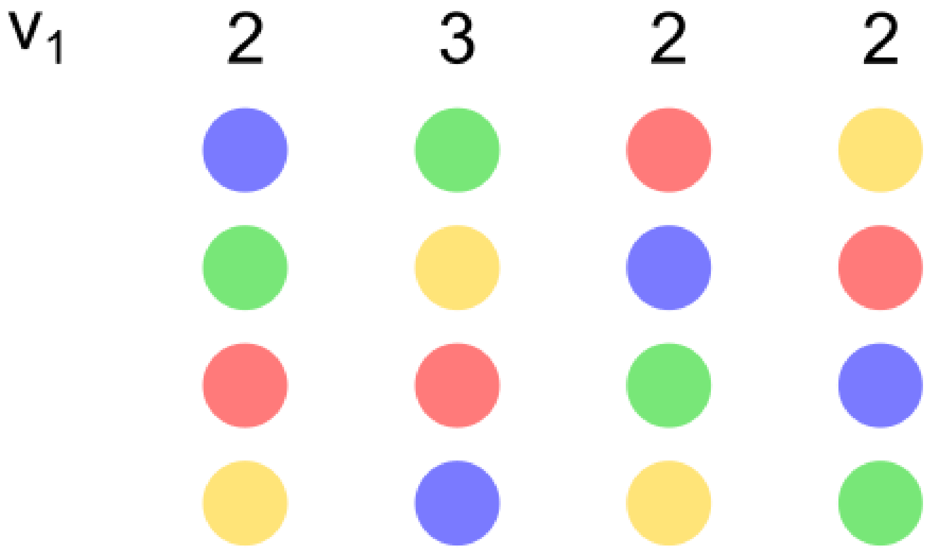


The greedy strategy does not always work

Manipulation under STV



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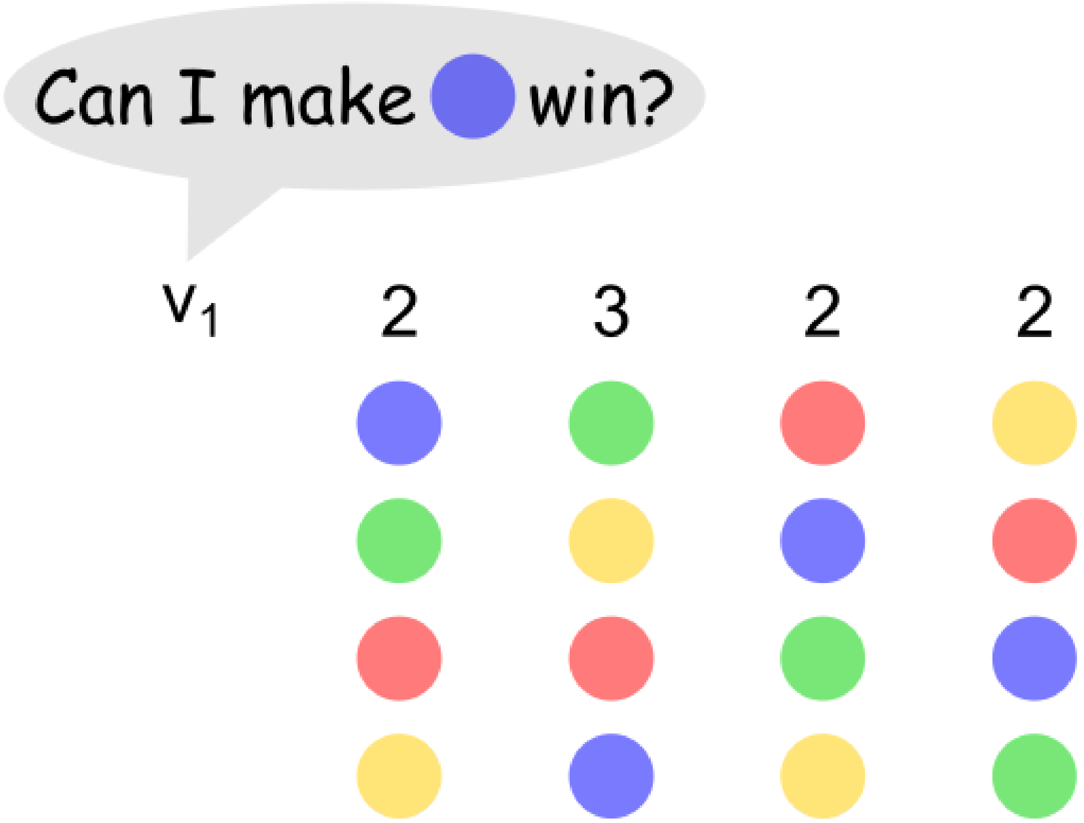

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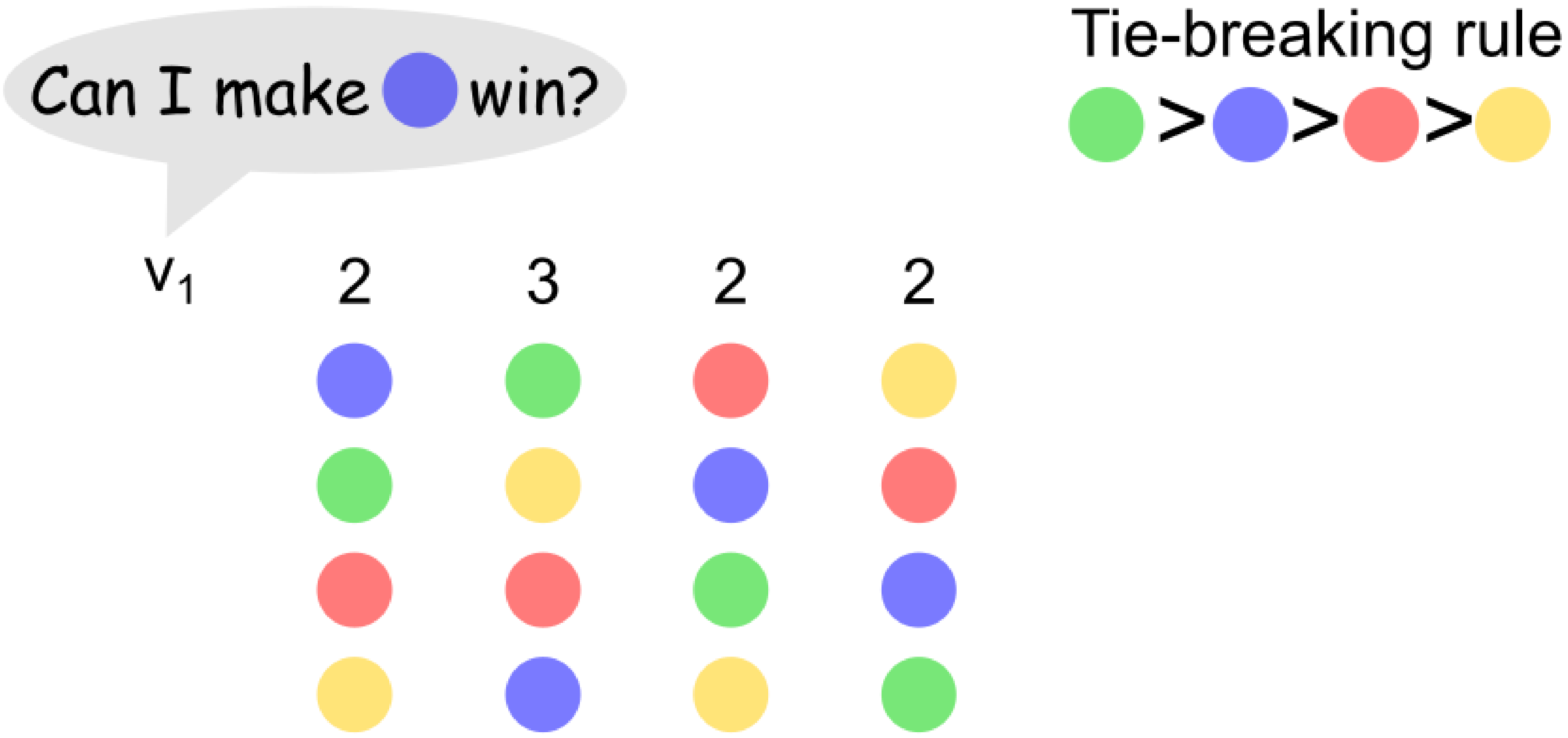

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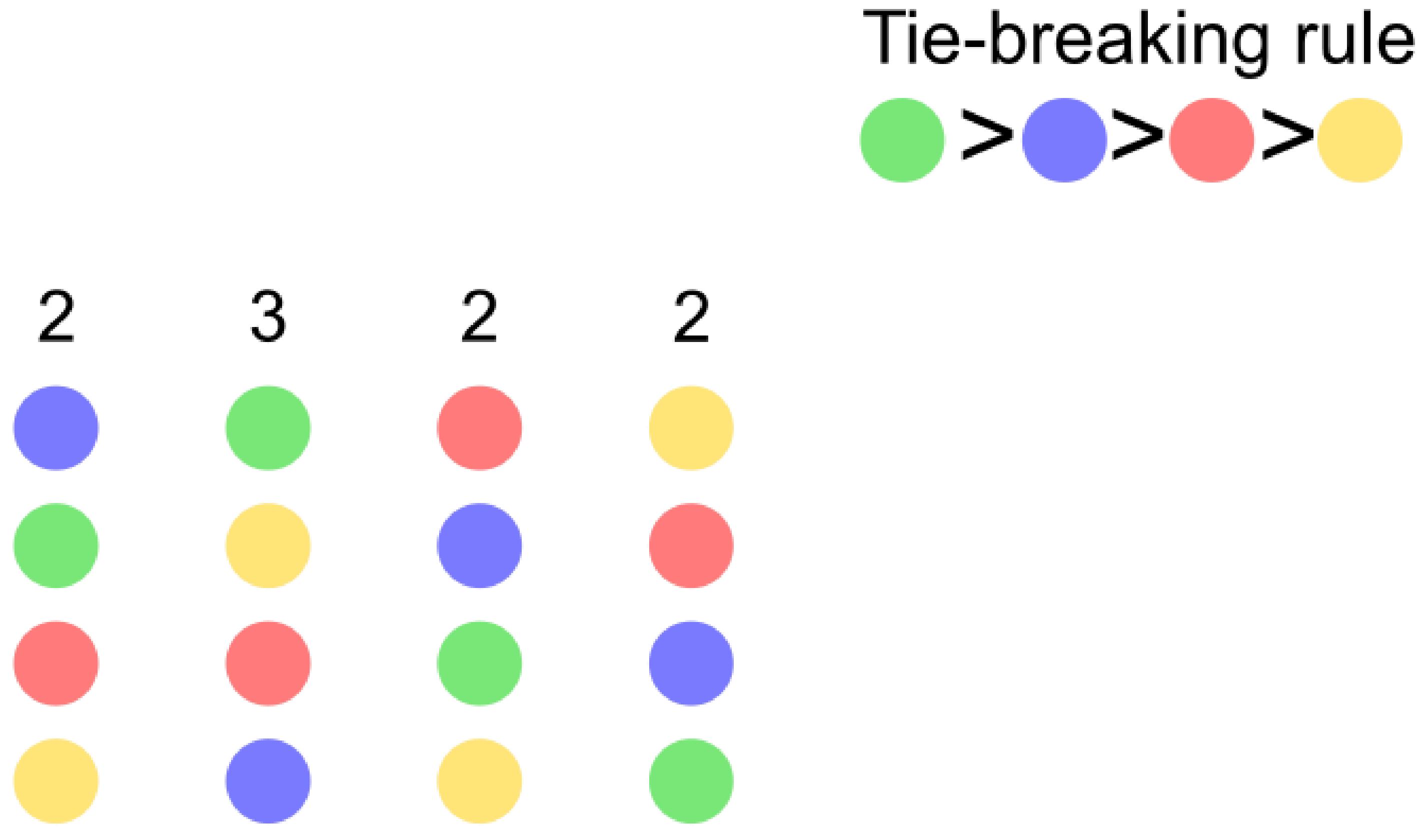
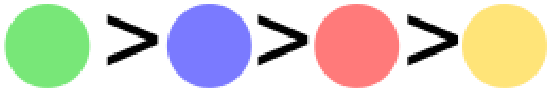
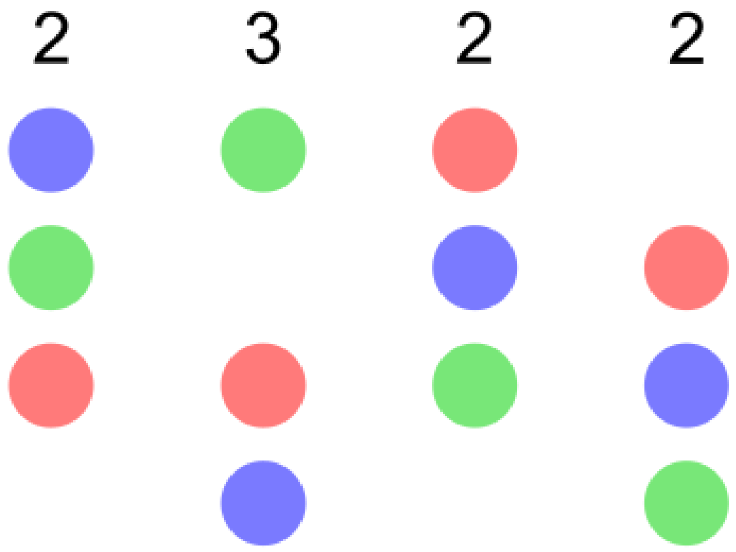


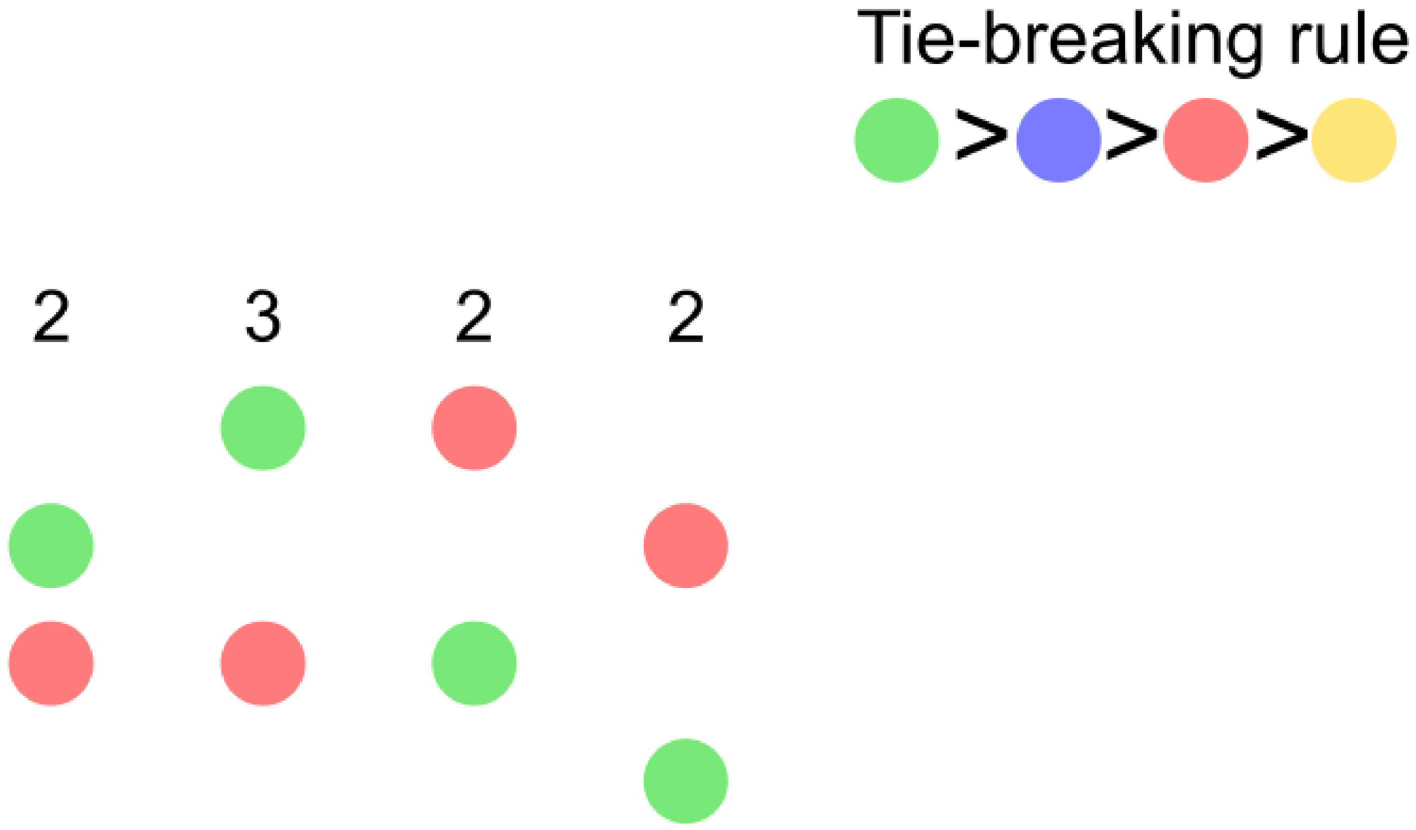
Manipulation under STV

Tie-breaking rule





Manipulation under STV



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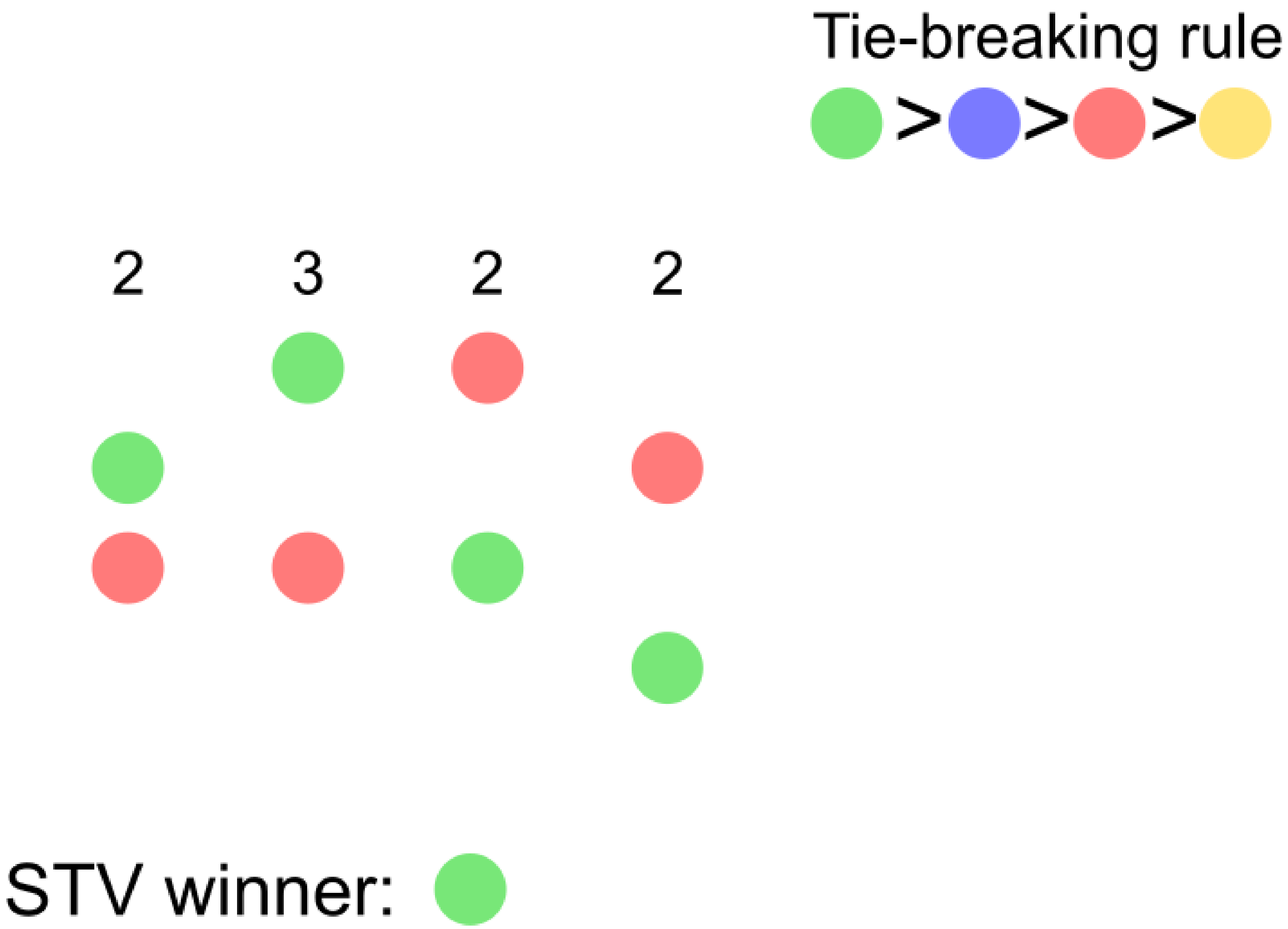

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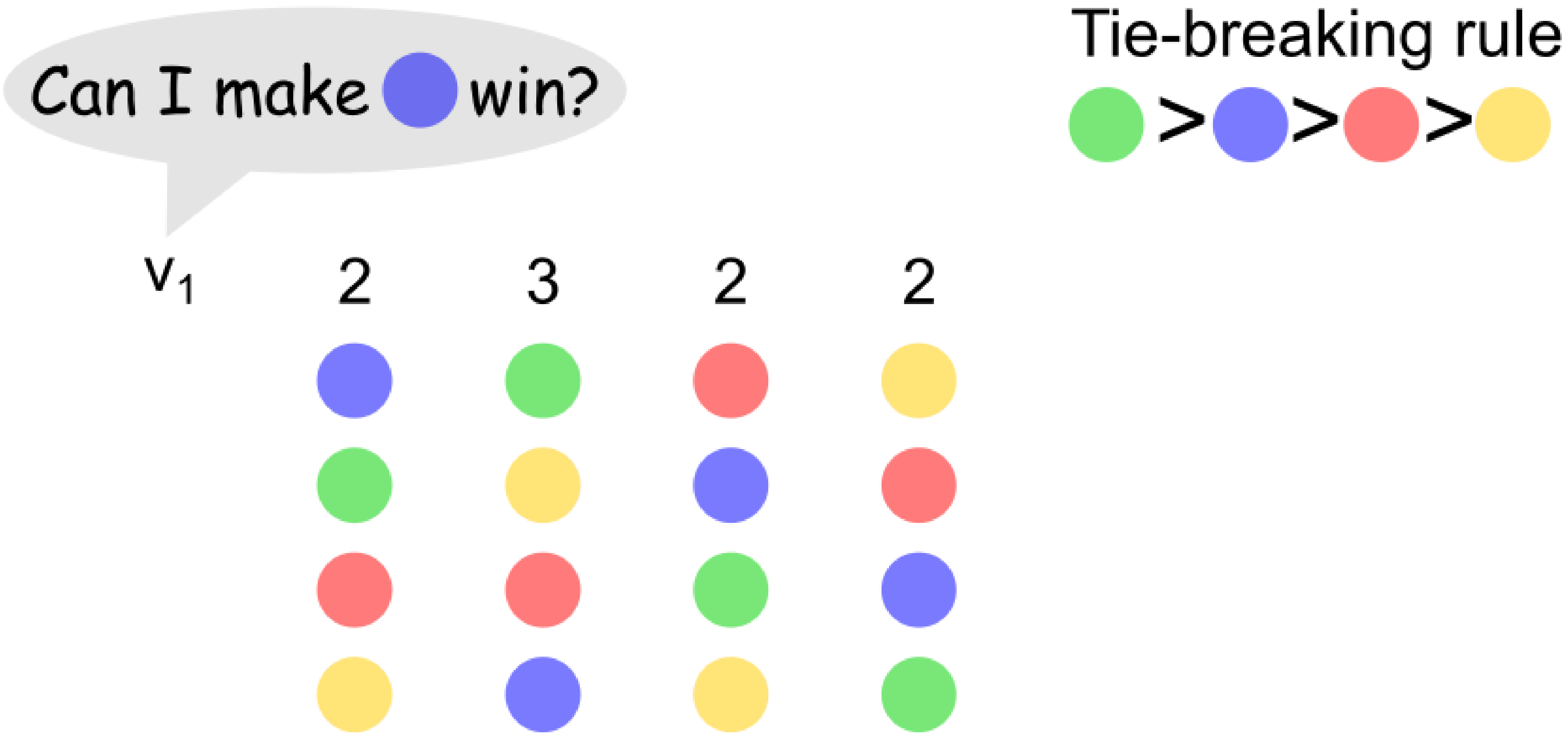

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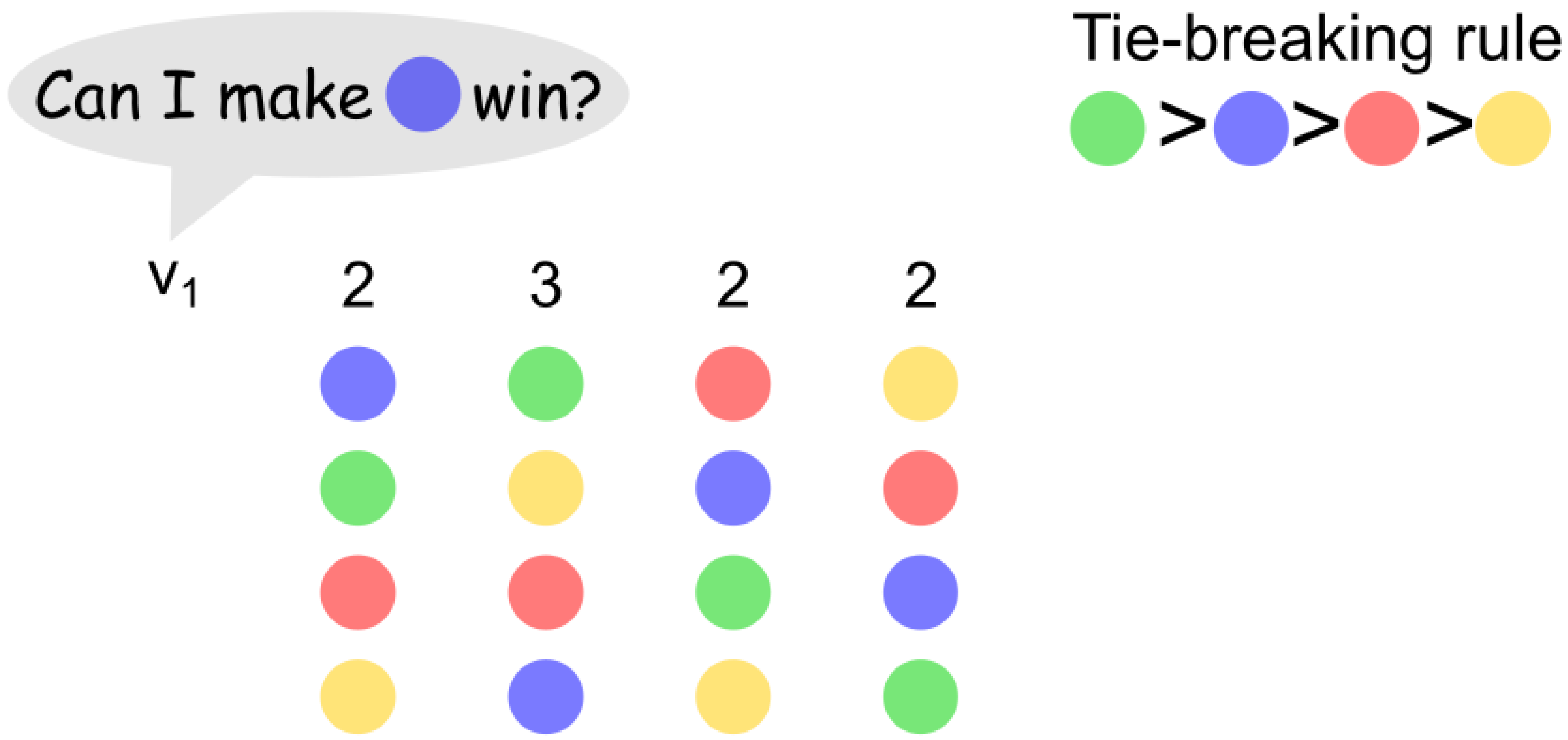
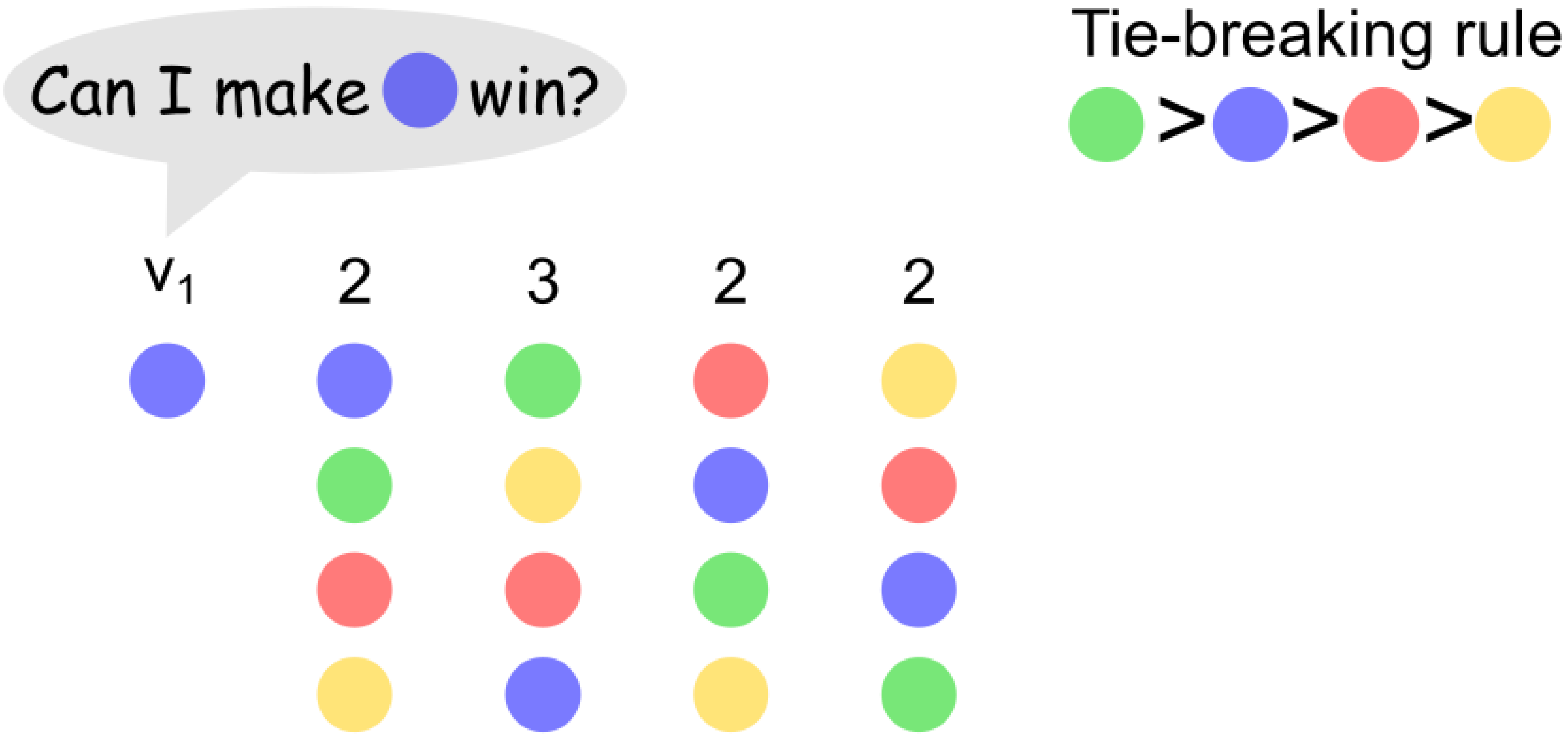


Let's follow the greedy strategy and put at the top.

Manipulation under STV



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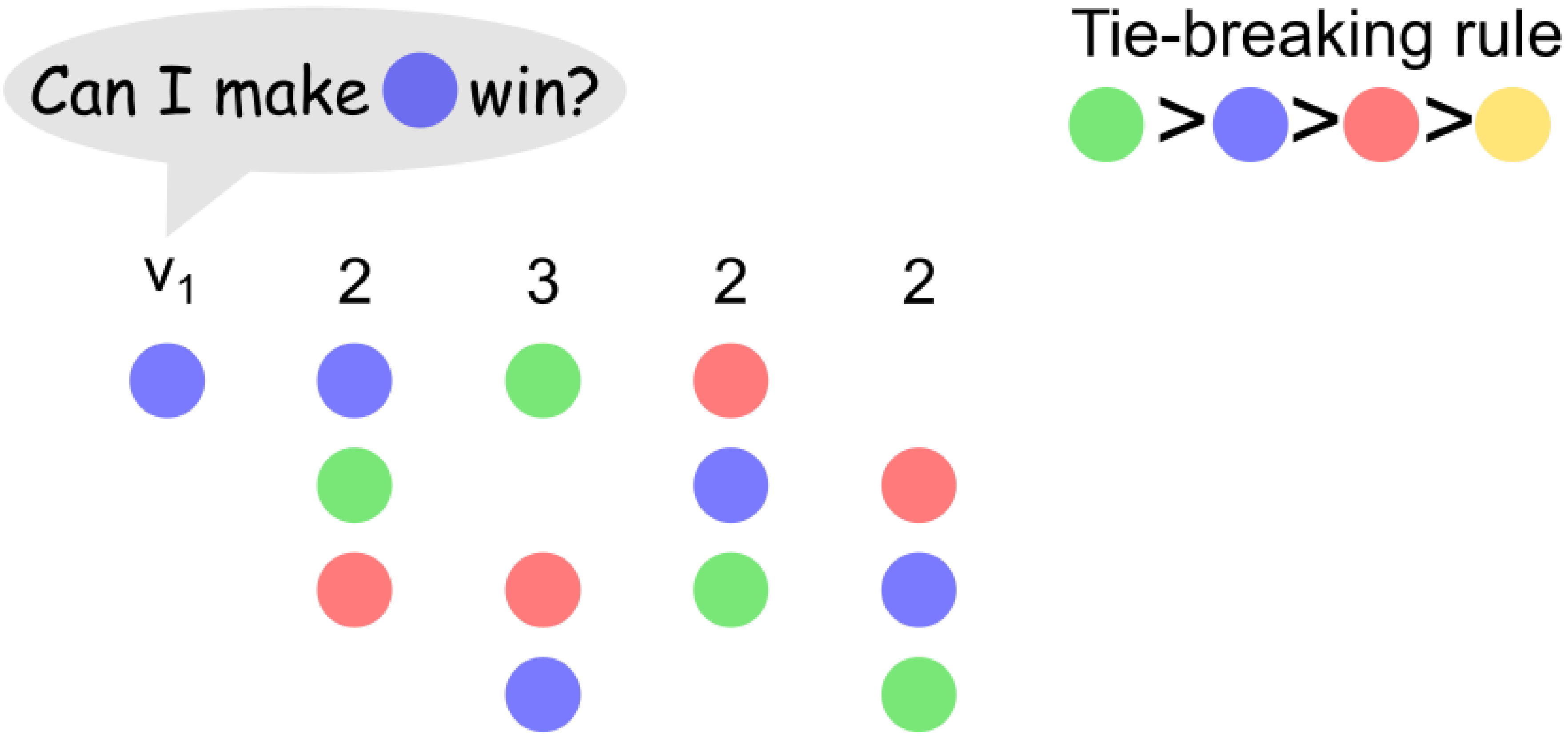

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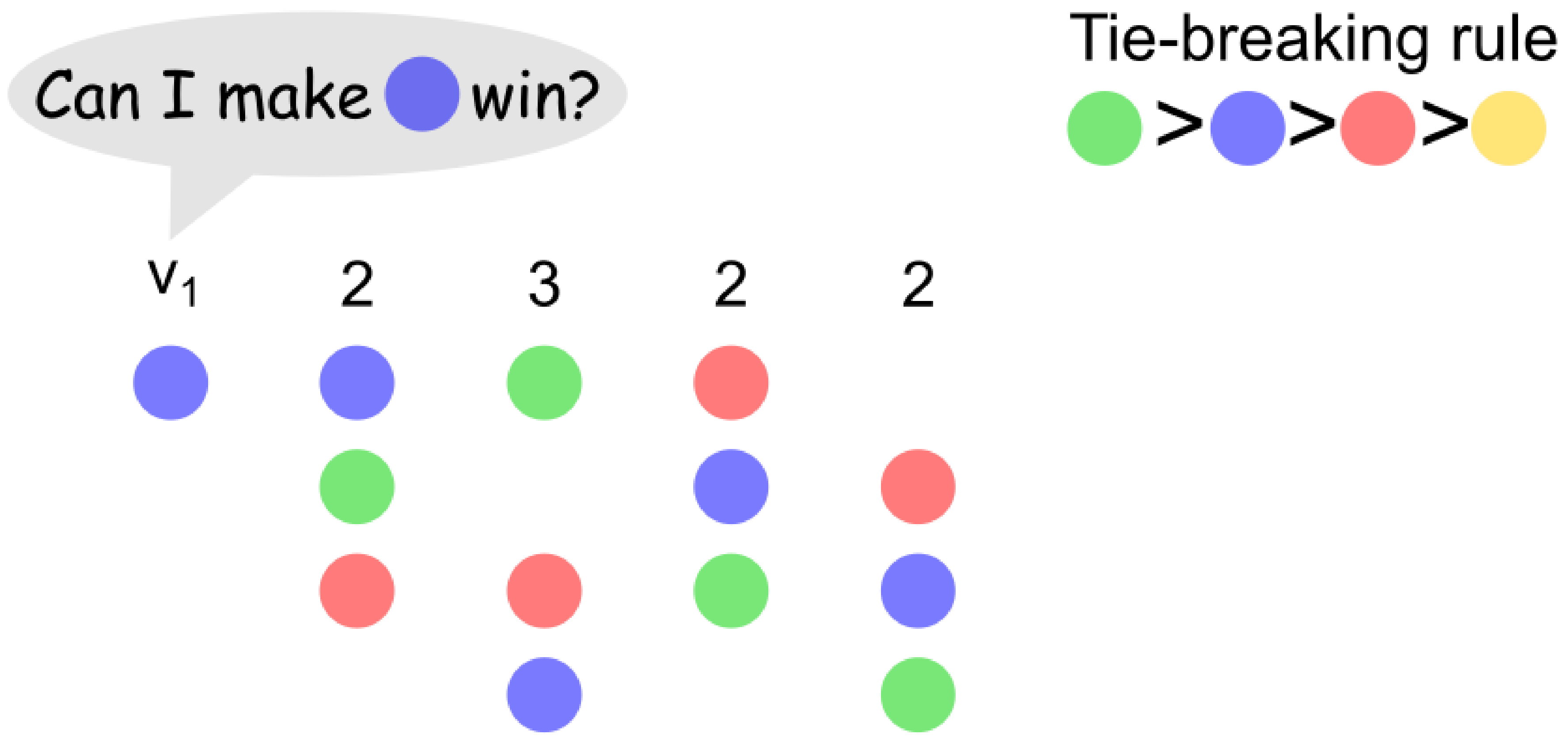

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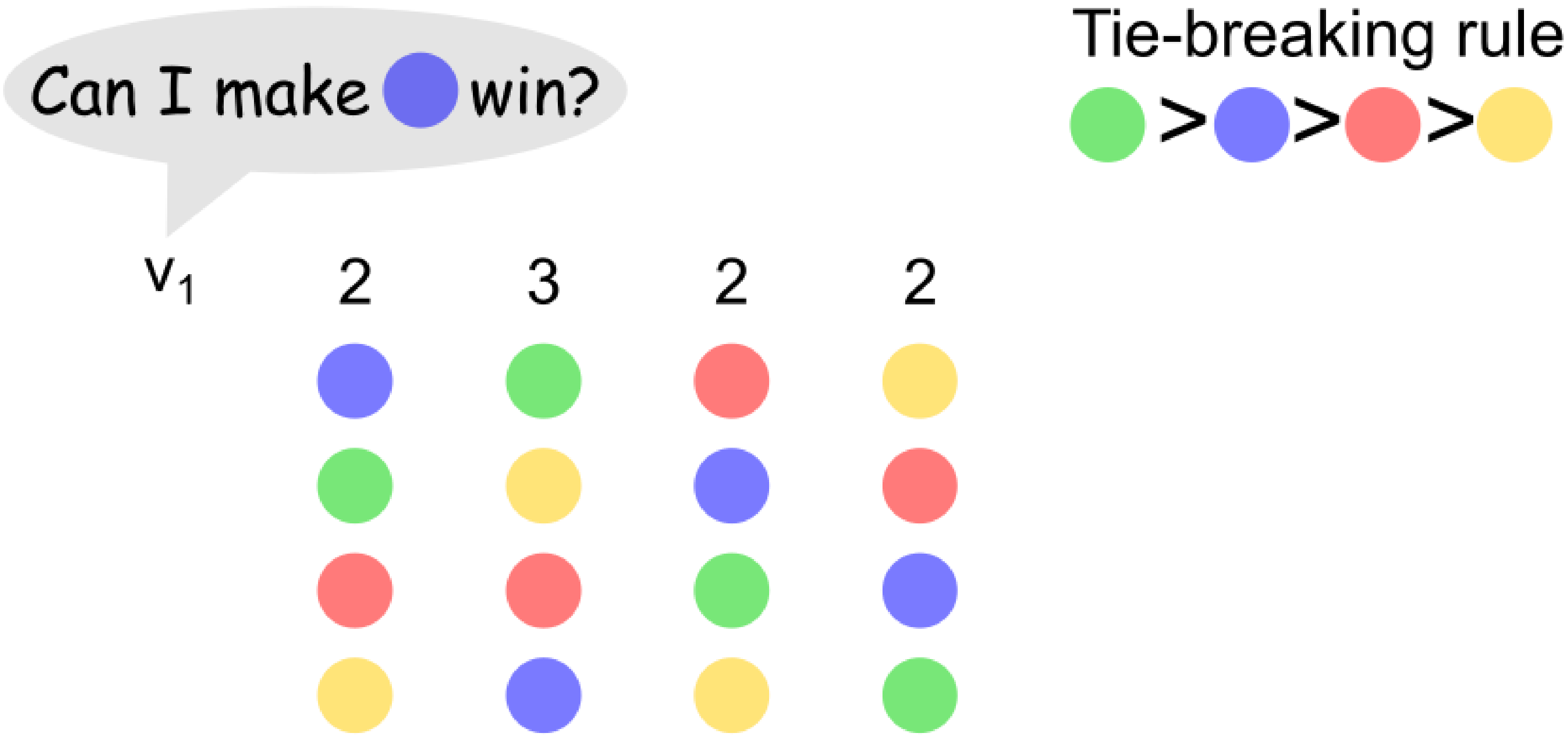
is eliminated in the next round (due to tie-breaking rule).

Manipulation under STV



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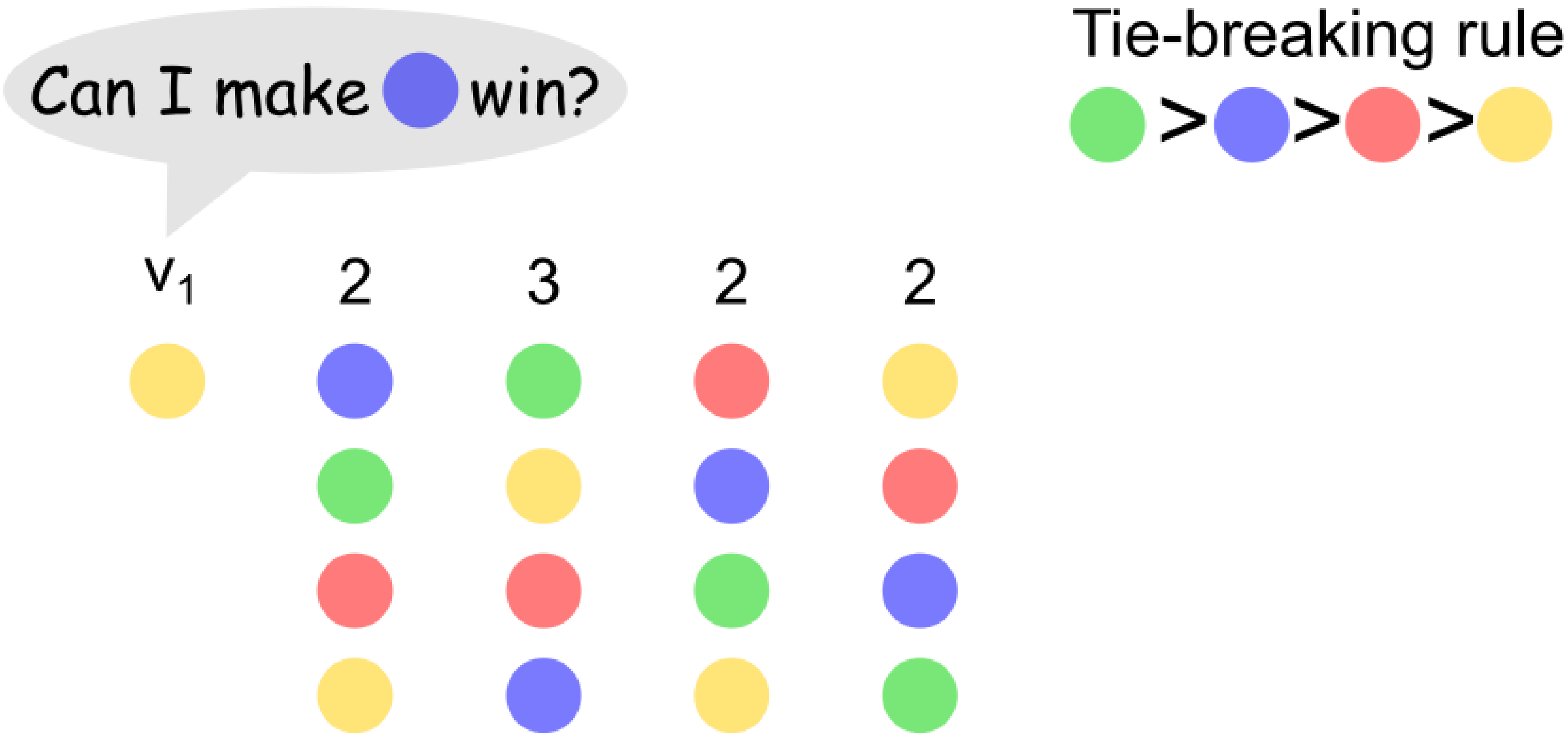
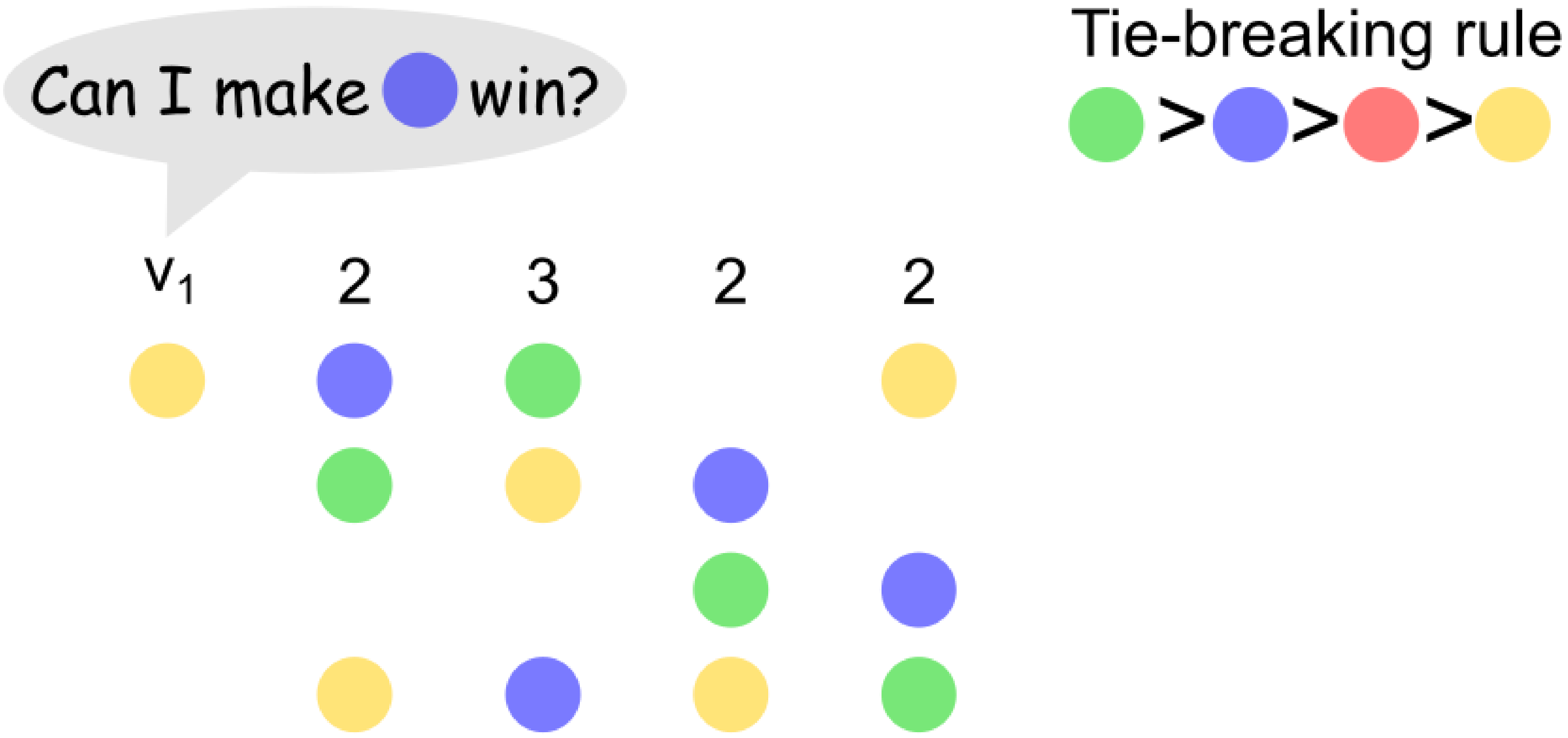


Manipulation under STV



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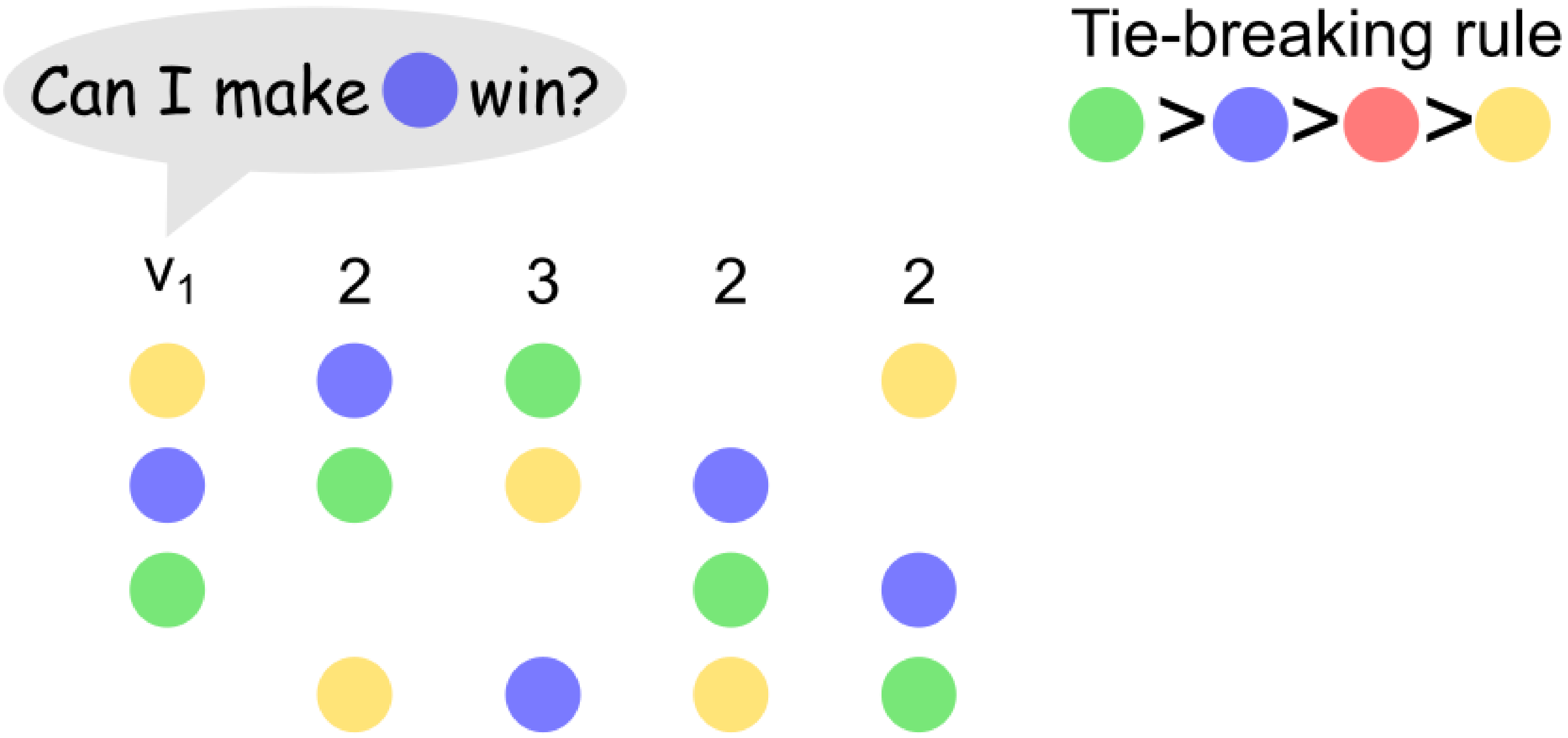

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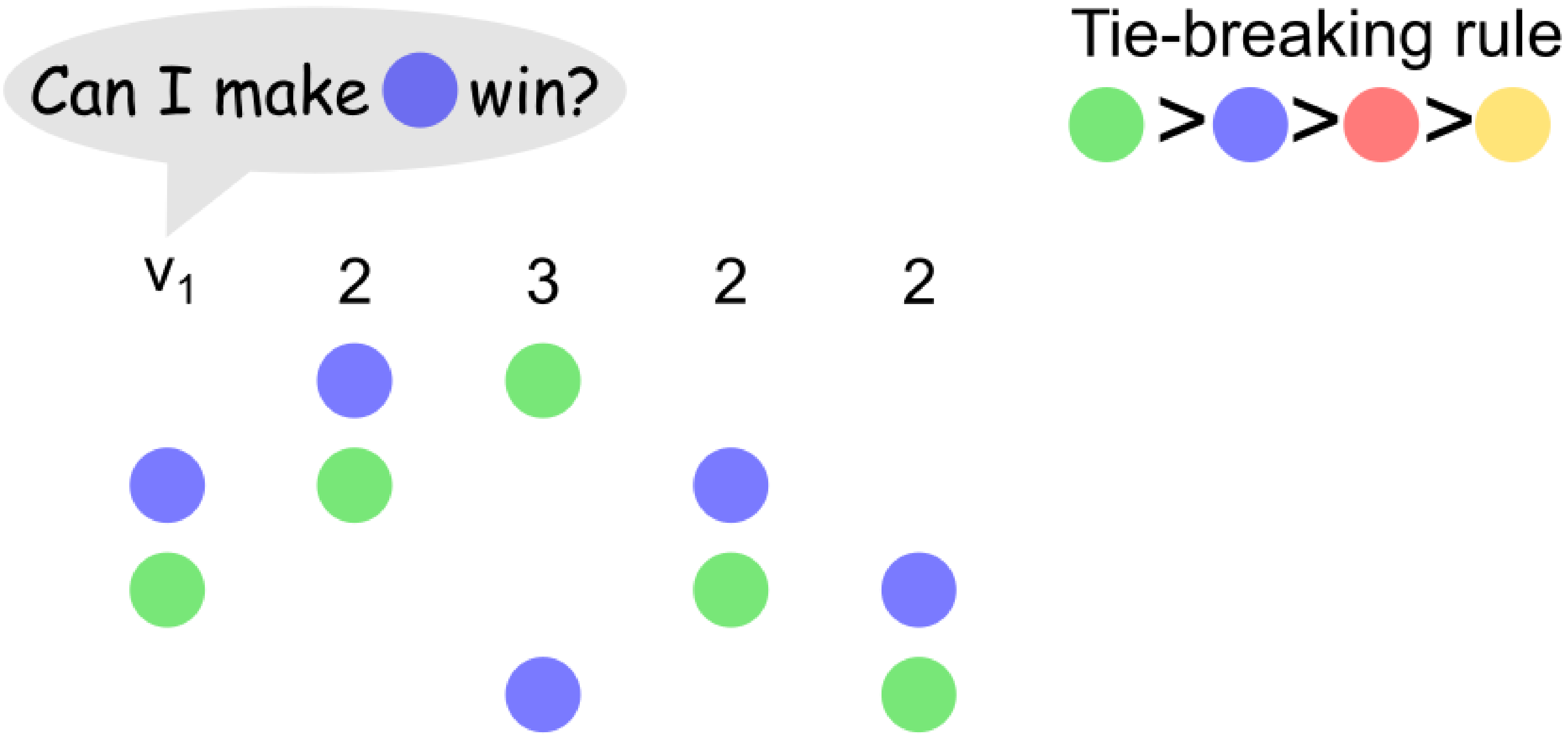
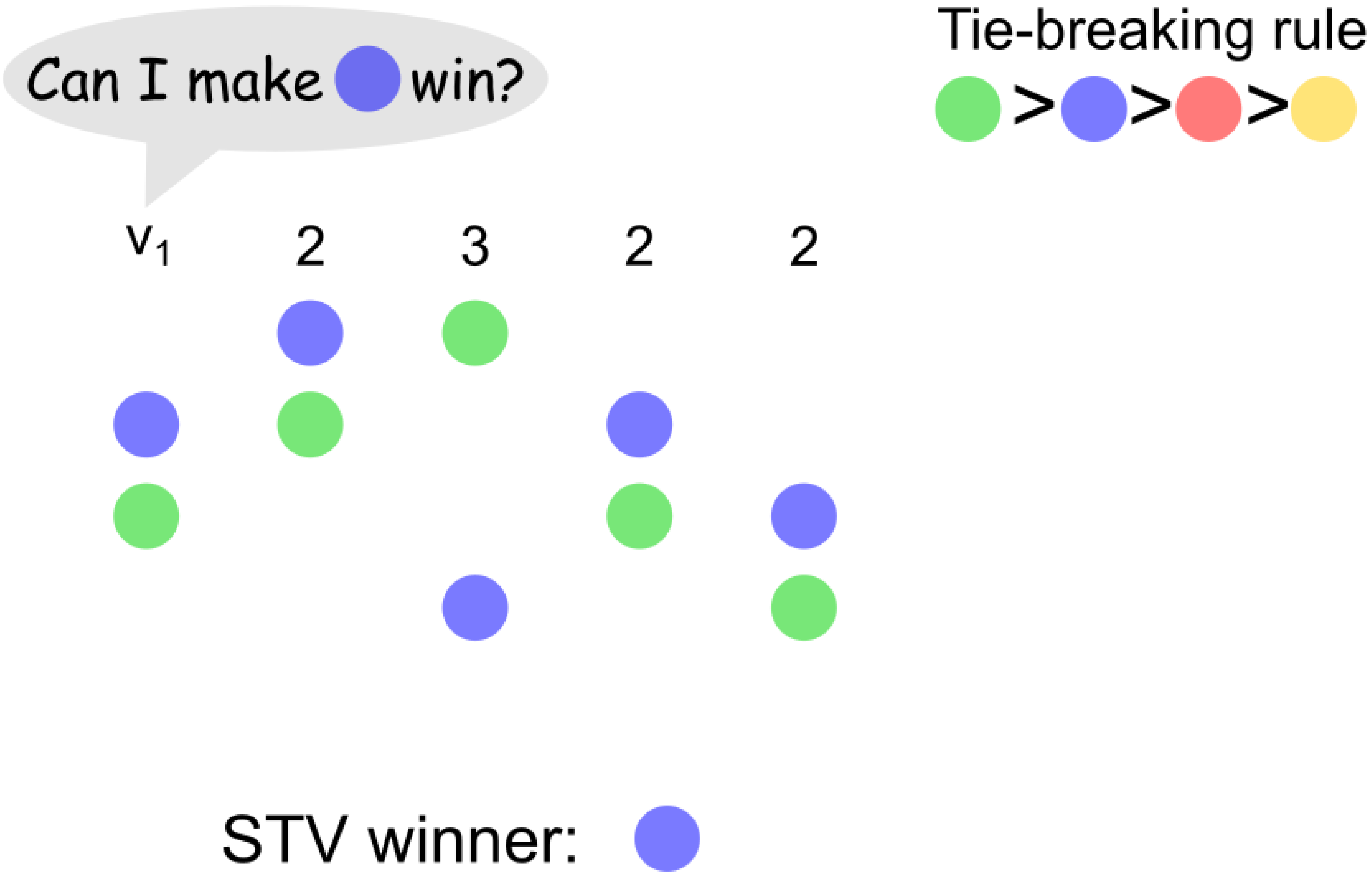


Manipulation under STV



So, when does the greedy strategy work?

[Bartholdi,Tovey and Trick, SCW 1989]

The greedy strategy can correctly solve f-Manipulation in polynomial time for any voting rule f satisfying:

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- Score-based: There exists a *scoring function* s: $(P_1, x) \rightarrow \mathbb{R}$ such that for any vote P_1 of v_1, the f-winner is the candidate maximizing $s(P_1, x)$.

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- **Efficiency**: The voting rule f can be evaluated in polynomial time.

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In particular, for f ∈ {Plurality, Borda, Copeland}.

Voting rule

Scoring function

Voting rule	Scoring function
Plurality	p_x = Plurality score of x from $P_2,\ldots,P_n$ $s(P_1,x) = \begin{cases} 1+p_x & \text{if x is top-ranked in } P_1 \\ p_x & \text{otherwise} \end{cases}$

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Borda	b_x = Borda score of x from $P_2,\ldots,P_n$ $s(P_1,x) = b_x$ + #candidates below x in P_1
Copeland	$s(P_1,x)$ = #candidates x beats in a head-to-head + 0.5.#candidates that x ties with in a head-to-head (based on all votes $P_1,P_2,\ldots,P_n$)

Correctness of Greedy Strategy

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Need to show:

If there is a winning vote for c, then the greedy strategy must also find one.

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W

x
k
c
d
•
•
s
b
q

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Let P be the partial list constructed by greedy before termination.

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q

P

c
x
q
s

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Consider the set of candidates that were not ranked by P. Among them, let k be ranked highest in W.

W	P
x	**c**
k	x
c	q
d	s
⋮	
s	
b	
q	

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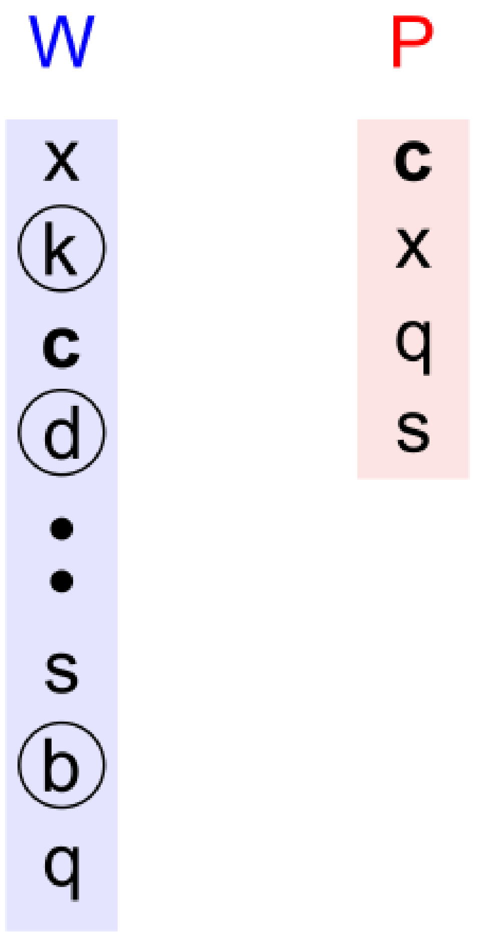

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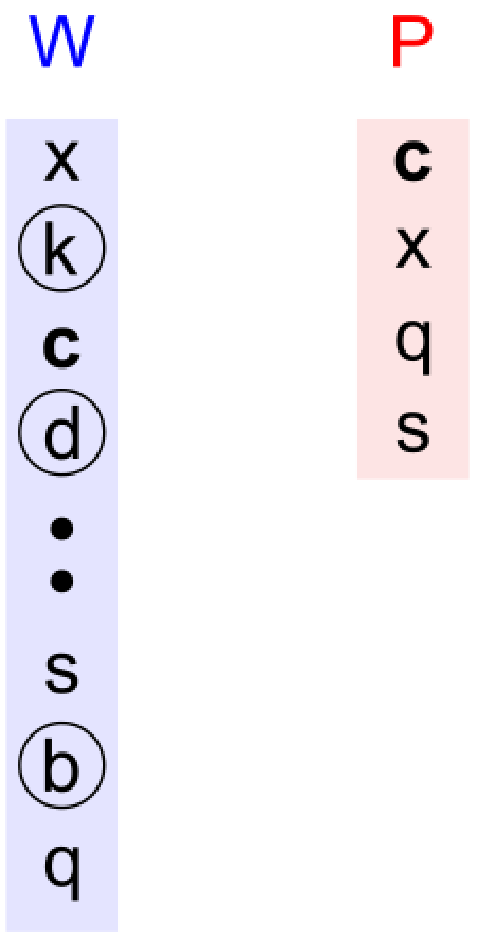

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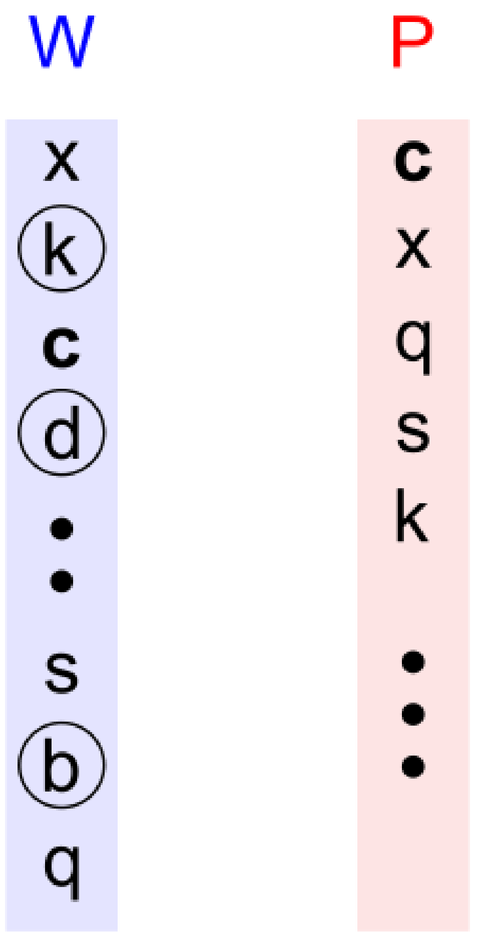

Correctness of Greedy Strategy

$s(P,\mathbf{c}) \geq s(W,\mathbf{c})$ by monotonicity of s

W	P
x	**c**
ⓚ	x
c	q
ⓓ	s
•	k
•	
	•
s	•
ⓑ	•
q	

Correctness of Greedy Strategy

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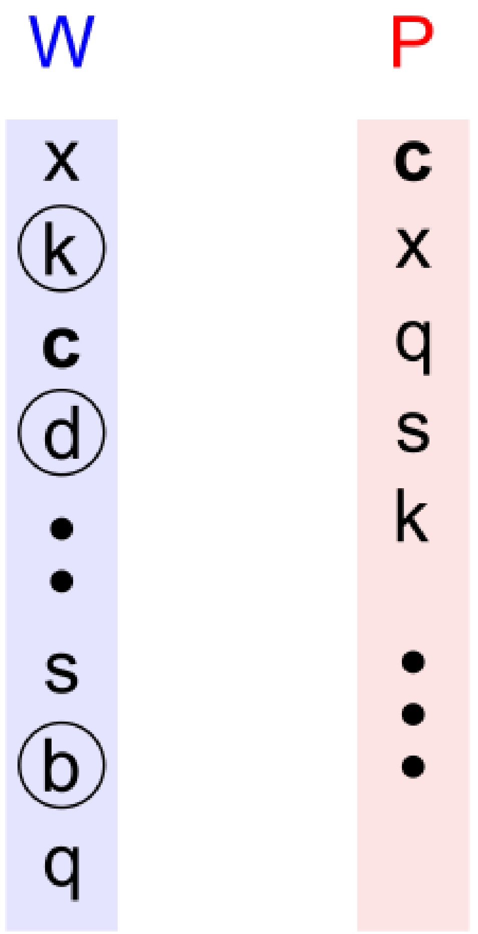
$s(W,\mathbf{c}) \ge s(W,k)$ since c wins under W



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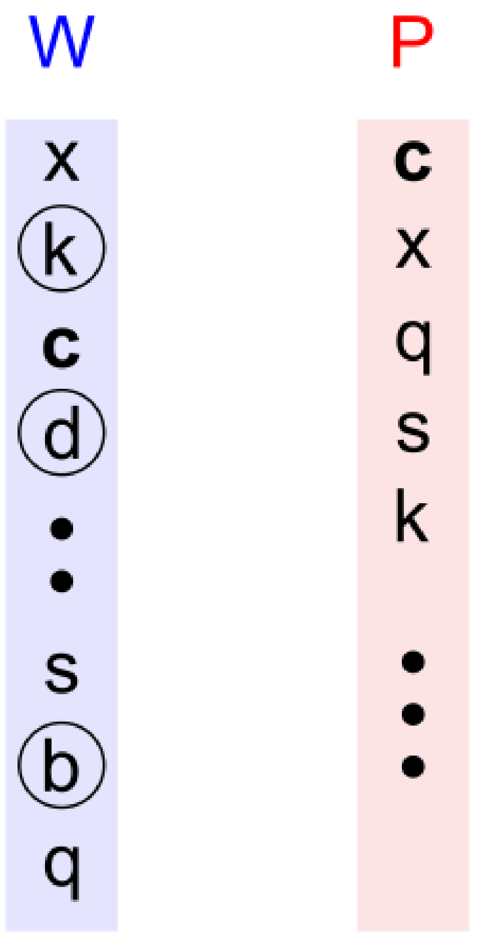
s(W,k) ≥ s(P,k) by monotonicity of s



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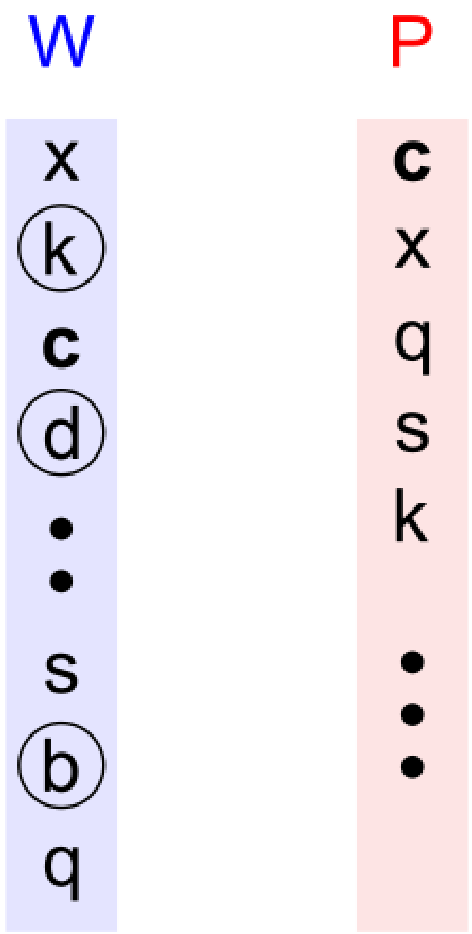

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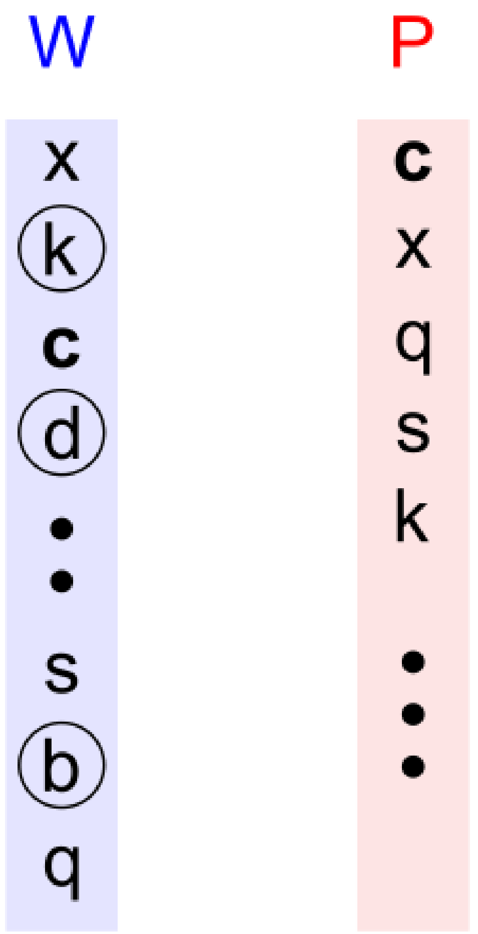

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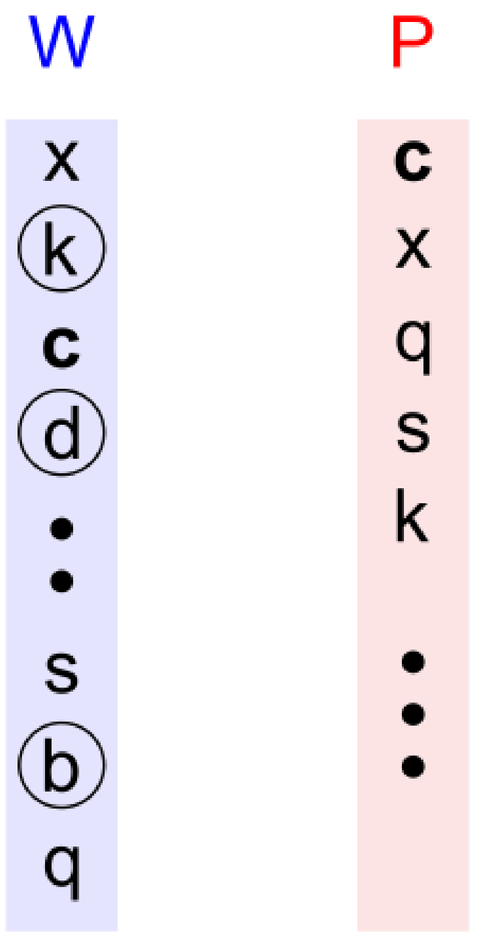

■

So, is manipulation always that easy?

Soc Choice Welfare (1989) 6: 227–241

Social Choice and Welfare

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The Computational Difficulty of Manipulating an Election*

J. J. Bartholdi III, C. A. Tovey, and M. A. Trick**

School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA

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Abstract. We show how computational complexity might protect the integrity of social choice. We exhibit a voting rule that efficiently computes winners but is computationally resistant to strategic manipulation. It is *NP*-complete for a manipulative voter to determine how to exploit knowledge of the preferences of others. In contrast, many standard voting schemes can be manipulated with only polynomial computational effort.

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Copeland with second-order tie-breaking

In case of a tie, winner is the candidate whose defeated competitors have the highest sum of Copeland scores.

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[Xia, Zuckerman, Procaccia, Conitzer, Rosenschein, IJCAI 2009]

Ranked Pairs

Consider candidate pairs according to the margin of head-to-head victories, and create a ranking based on it while avoiding cycles.

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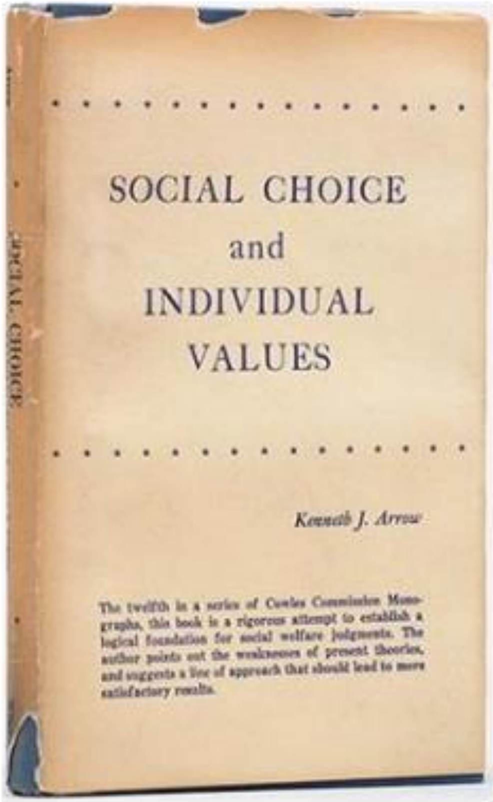
Note: NP-hard *even with* full information.

Remember this?

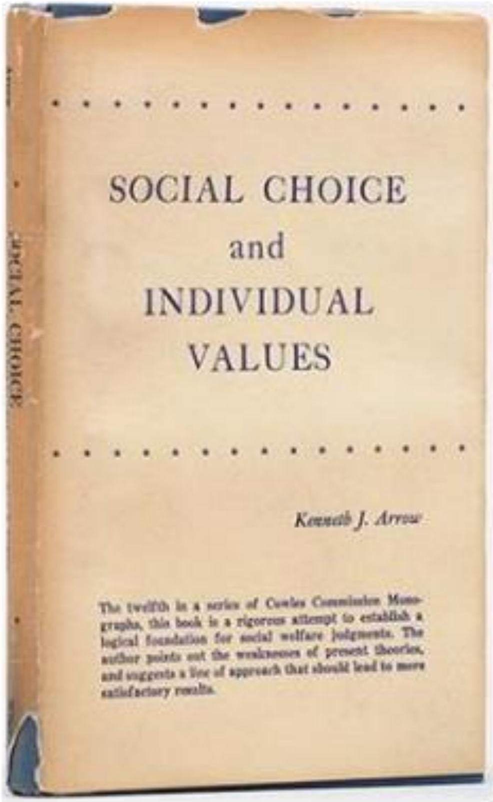
Method \ Criterion	Majority	Maj. loser	Mutual maj.	Condorcet	Cond. loser	Smith/ ISDA	LIIA	IIA	Cloneproof	Monotone	Consistency	Participation	Reversal symmetry	Polytime/ resolvable		Summable	Later-no-		No favorite betrayal	Ballot type	Ranks	
																	Harm	Help			=	>2
Approval	Rated[a]	No	No	No[b][c]	No	No[b]	Yes	Yes[d]	Yes[e]	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes[f]	Yes	Approvals	Yes	No
Borda count	No	Yes	No	No[b]	Yes	No	No	No	Teams	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	No	Ranking	Yes	Yes
Bucklin	Yes	Yes	Yes	No	No	No	No	No	No	Yes	No	No	No	O(N)	Yes	O(N)	No	Yes	If equal preferences	Ranking	Yes	Yes
Copeland	Yes	Yes	Yes	Yes	Yes	Yes	No	No[b]	Teams, crowds	Yes	No[b]	No[b]	Yes	$O(N^2)$	No	$O(N^2)$	No[b]	No	No[b]	Ranking	Yes	Yes
IRV (AV)	Yes	Yes	Yes	No[b]	Yes	No[b]	No	No	Yes	No	No	No	No	$O(N^2)$	Yes[g]	$O(N!)$[h]	Yes	Yes	No	Ranking	No	Yes
Kemeny–Young	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No[b]	Spoilers	Yes	No[b] [i]	No[b]	Yes	$O(N!)$	Yes	$O(N^2)$[j]	No[b]	No	No[b]	Ranking	Yes	Yes
Highest median/Majority judgment[k]	Rated[l]	Yes[m]	No[n]	No[b][c]	No	No[b]	Yes	Yes[d]	Yes	Yes	No[o]	No[p]	Depends[q]	O(N)	Yes	O(N)[r]	No[s]	Yes	Yes	Scores[t]	Yes	Yes
Minimax	Yes	No	No	Yes[u]	No	No	No	No[b]	Spoilers	Yes	No[b]	No[b]	No	$O(N^2)$	Yes	$O(N^2)$	No[b][u]	No	No[b]	Ranking	Yes	Yes
Plurality/FPTP	Yes	No	No	No[b]	No	No[b]	No	No	Spoilers	Yes	Yes	Yes	No	O(N)	Yes	O(N)	N/A[v]	N/A[v]	No	Single mark	N/A	No
Score voting	No	No	No	No[b][c]	No	No[b]	Yes	Yes[d]	Yes	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	Yes	Scores	Yes	Yes
Ranked pairs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No[b]	Yes	Yes	No[b]	No[p][b]	Yes	$O(N^3)$	Yes	$O(N^2)$	No[b]	No	No[p][b]	Ranking	Yes	Yes
Runoff voting	Yes	Yes	No	No[b]	Yes	No[b]	No	No	Spoilers	No	No	No	No	O(N)[w]	Yes	O(N)[w]	Yes	Yes[x]	No	Single mark	N/A	No[y]
Schulze	Yes	Yes	Yes	Yes	Yes	Yes	No	No[b]	Yes	Yes	No[b]	No[p][b]	Yes	$O(N^3)$	Yes	$O(N^2)$	No[b]	No	No[p][b]	Ranking	Yes	Yes
STAR voting	No[z]	Yes	No[aa]	No[b][c]	Yes	No[b]	No	No	No	Yes	No	No	Depends[ab]	O(N)	Yes	$O(N^2)$	No	No	No[ac]	Scores	Yes	Yes
Sortition, arbitrary winner[ad]	No	No	No	No[b]	No	No[b]	Yes	Yes	No	Yes	Yes	Yes	Yes	O(1)	No	O(1)	Yes	Yes	Yes	None	N/A	N/A
Random ballot[ae]	No	No	No	No[b]	No	No[b]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	O(N)	No	O(N)	Yes	Yes	Yes	Single mark	N/A	No

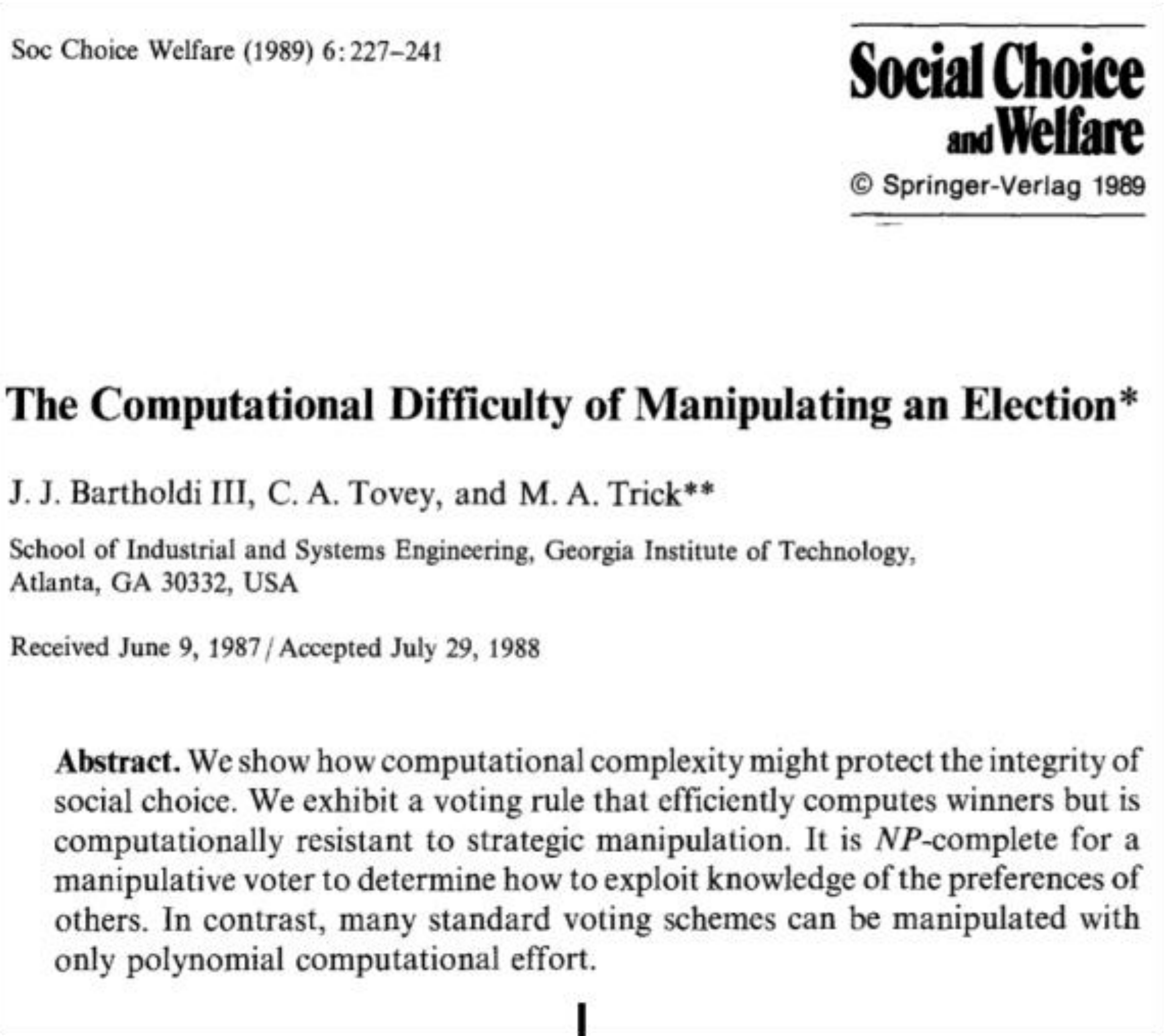
	Single manipulator
Plurality	P [Bartholdi, Tovey and Trick, SCW 1989]
Borda	P [Bartholdi, Tovey and Trick, SCW 1989]
Copeland$^{\alpha}$ (friendly tie-breaking)	P [Bartholdi, Tovey and Trick, SCW 1989]
Ranked pairs	NP-hard [Xia, Zuckerman, Procaccia, Conitzer, and Rosenschein, IJCAI 2009]
Schulze	P [Parkes and Xia, AAAI 2012]

	Single manipulator	Two manipulators
Plurality	P [Bartholdi, Tovey and Trick, SCW 1989]	P
Borda	P [Bartholdi, Tovey and Trick, SCW 1989]	NP-hard [Betzler, Niedermeier and Woeginger, IJCAI 2011; Davies, Katsirelos, Narodytska and Walsh, AAAI 2011]
Copeland$^{\alpha}$ (friendly tie-breaking)	P [Bartholdi, Tovey and Trick, SCW 1989]	NP-hard [Faliszewski, Hemaspaandra and Schnoor, AAMAS 2008]
Ranked pairs	NP-hard [Xia, Zuckerman, Procaccia, Conitzer, and Rosenschein, IJCAI 2009]	NP-hard [Xia, Zuckerman, Procaccia, Conitzer, and Rosenschein, IJCAI 2009]
Schulze	P [Parkes and Xia, AAAI 2012]	P [Gaspers, Kalinowski, Narodytska and Walsh, AAMAS 2013]



Social Choice Theory





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Social Choice Theory

Computational Social Choice

Enough about voting. Let’s talk sports!

ELIMINATION IN SPORTS

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Imagine we are at the halfway point of a sports tournament.

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Some games have been played, others are still to go.

ELIMINATION IN SPORTS

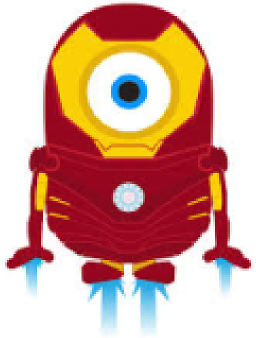
Imagine we are at the halfway point of a sports tournament.

Some games have been played, others are still to go.

Q: Does my favorite team still have a chance of winning?







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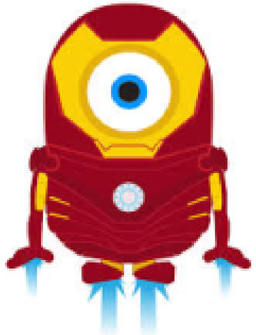

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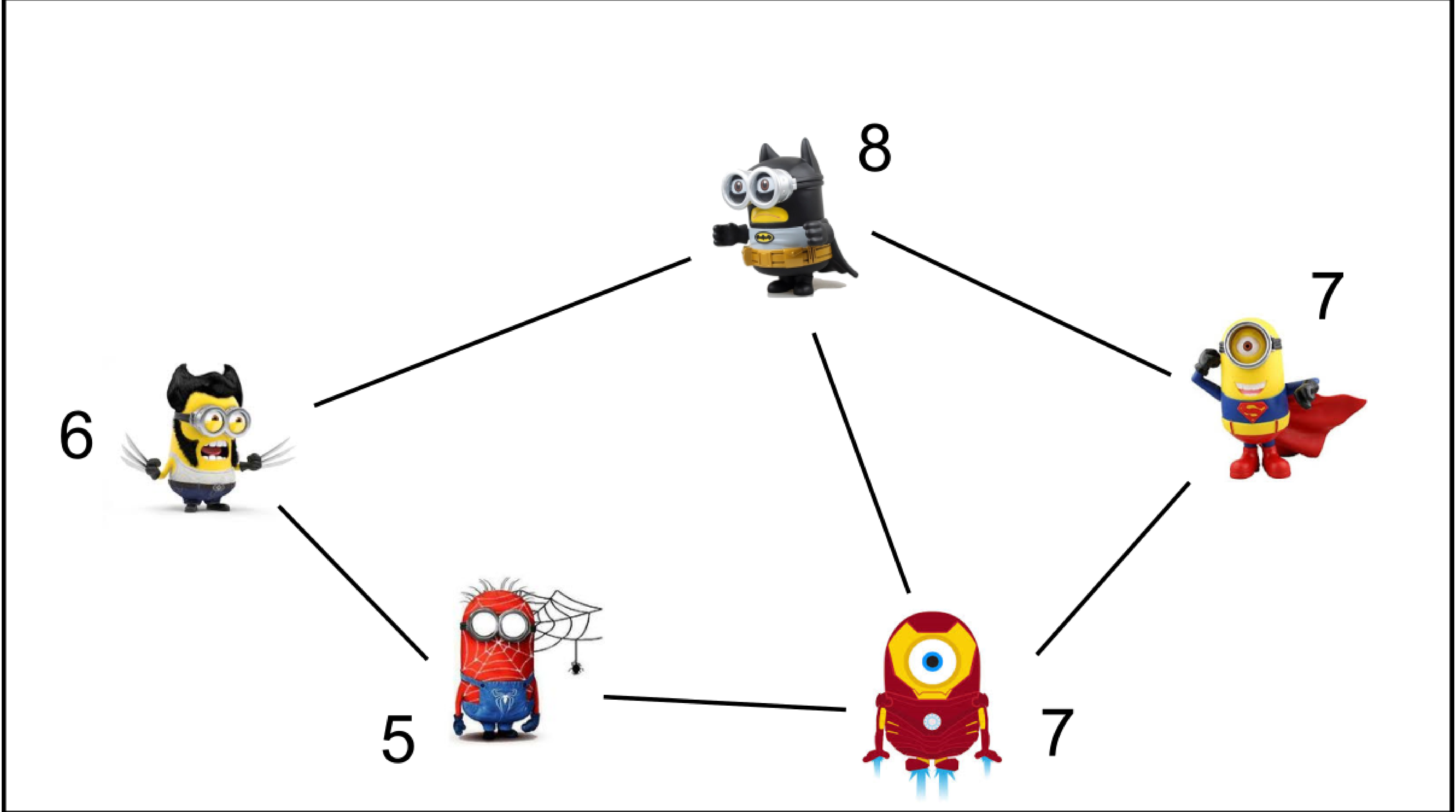
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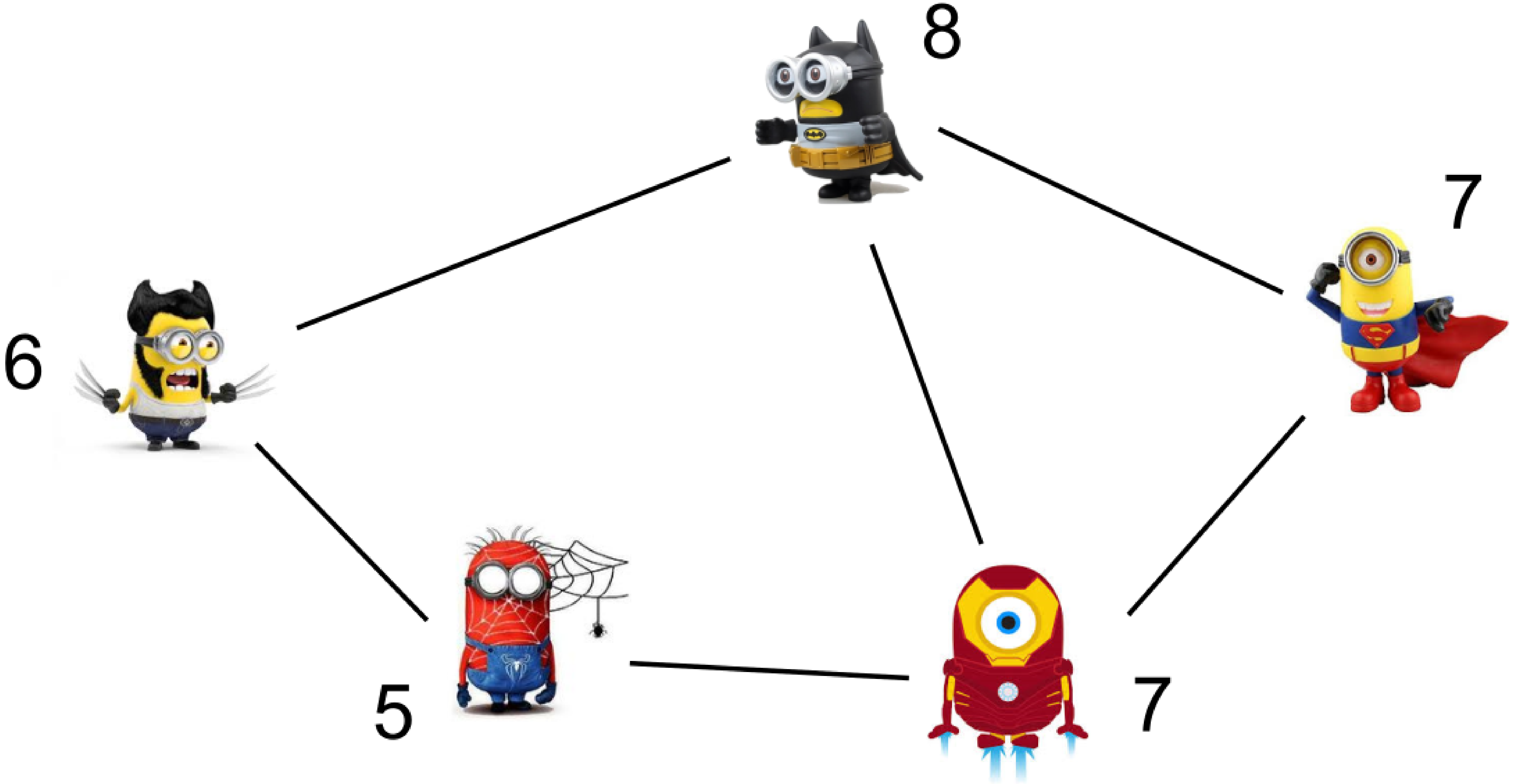
7

After each game, winner gets 1 point, loser get 0. No ties.



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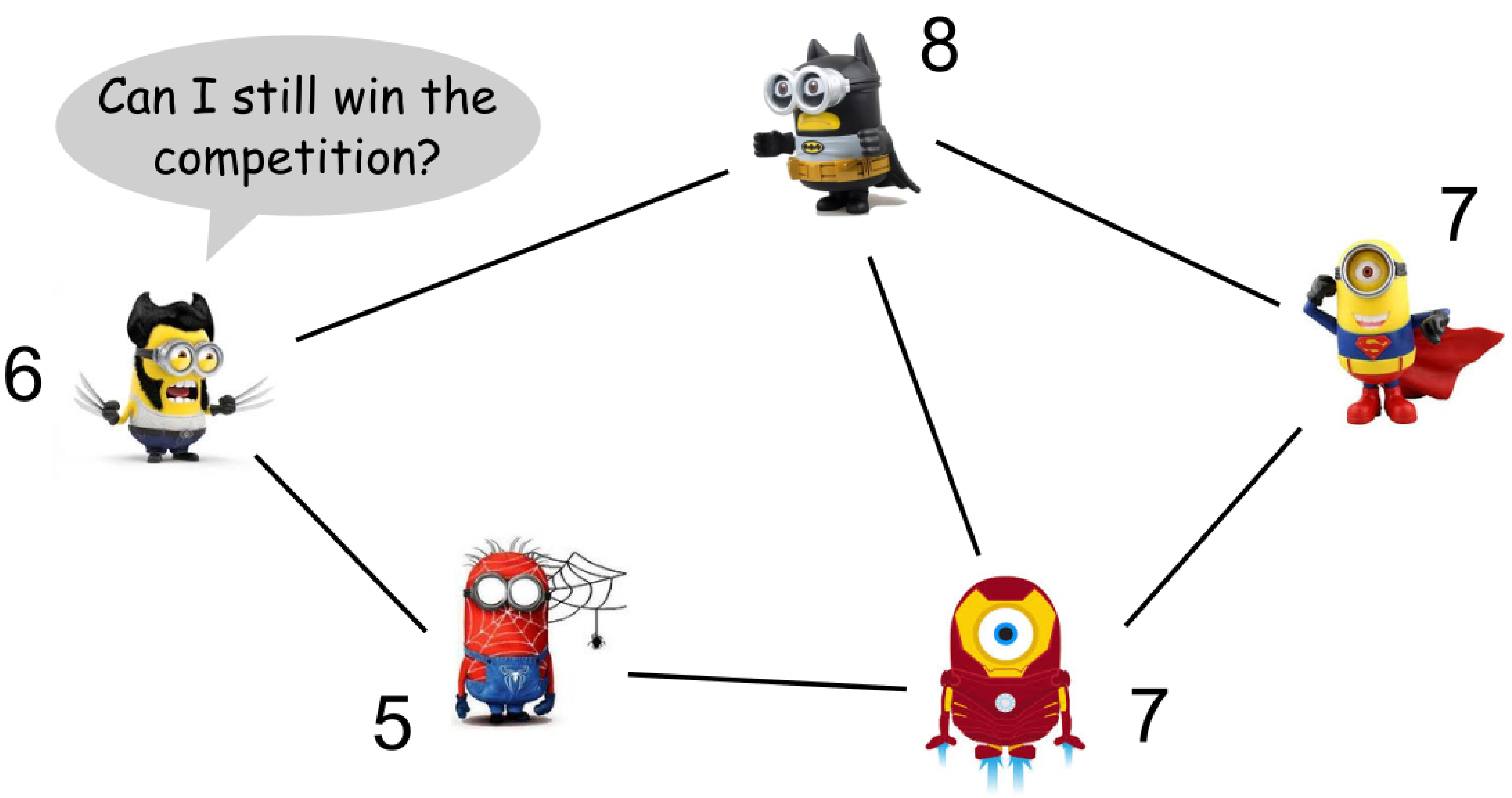

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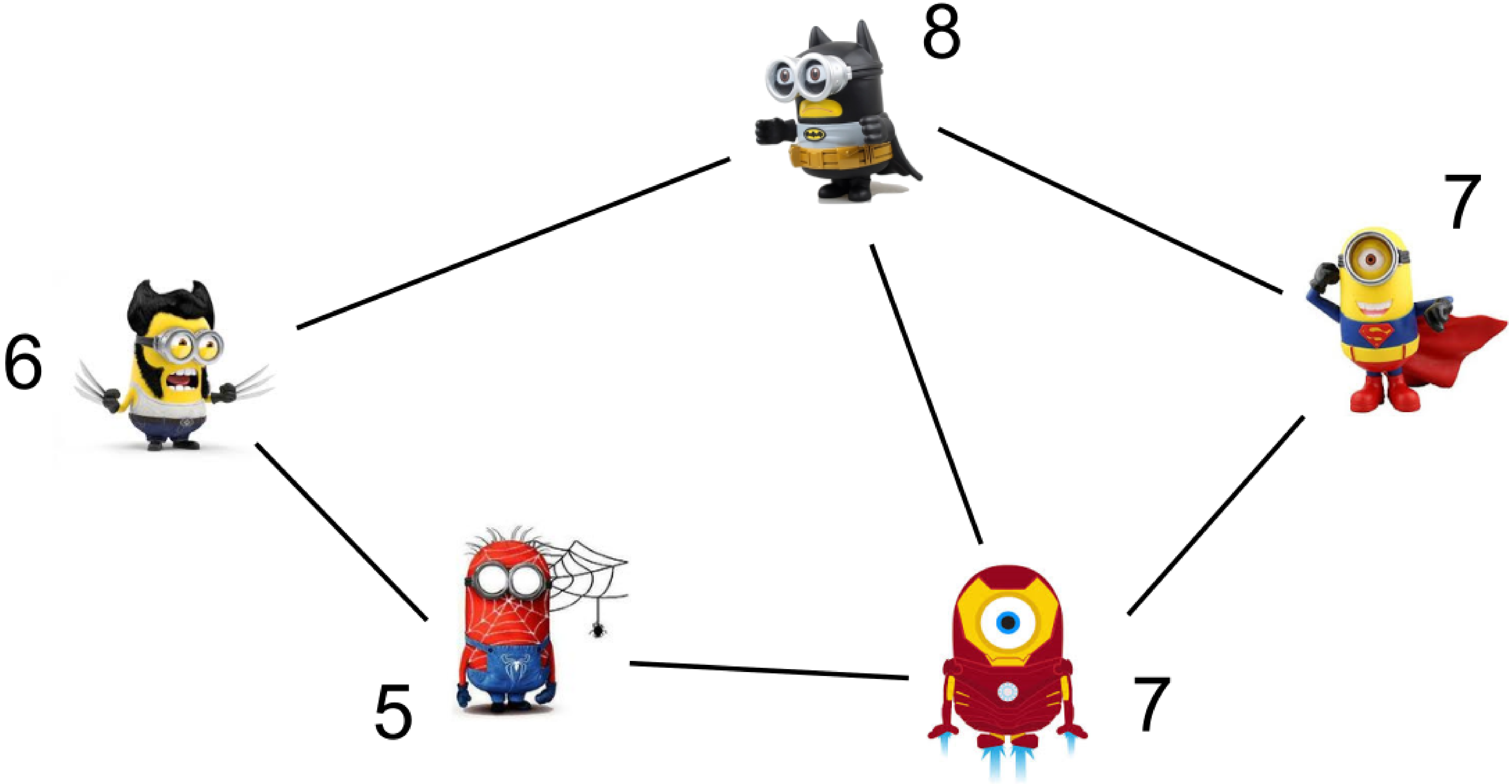

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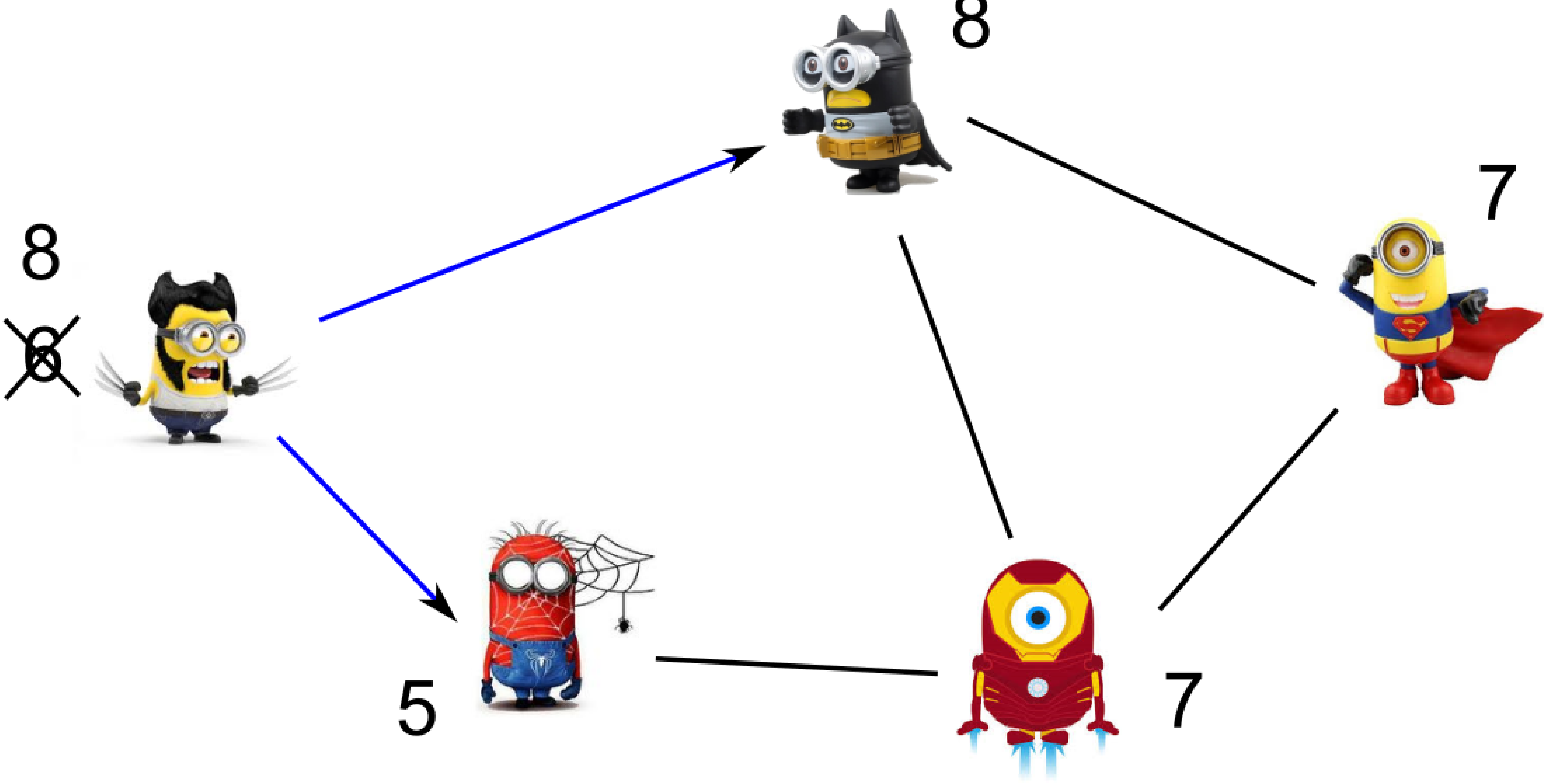

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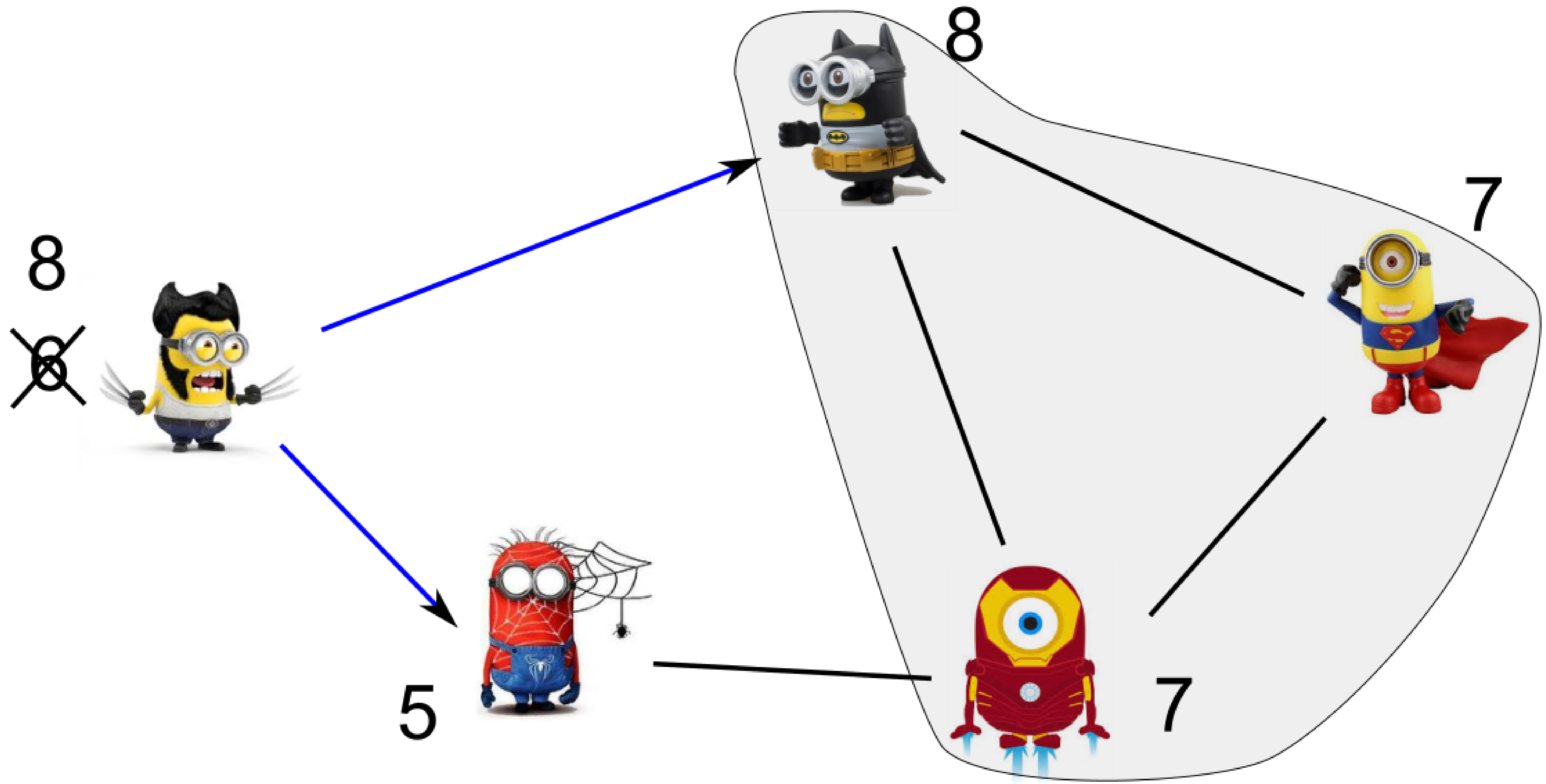

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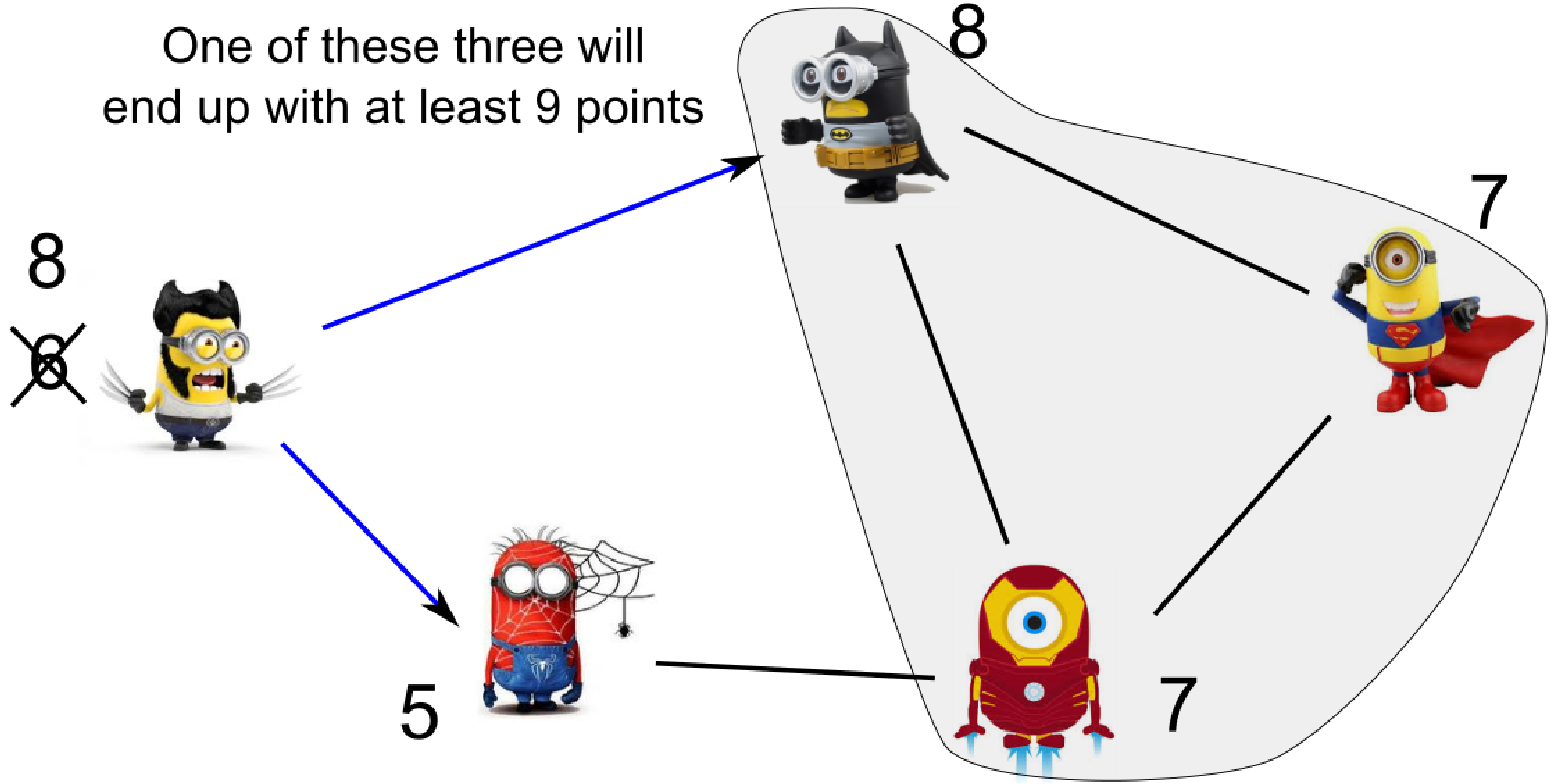

We will solve this problem using max flow.

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Step 1: Imagine  wins all its remaining games.

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Doing so freezes the score of 

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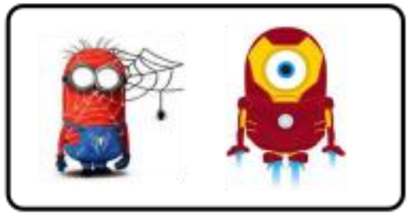
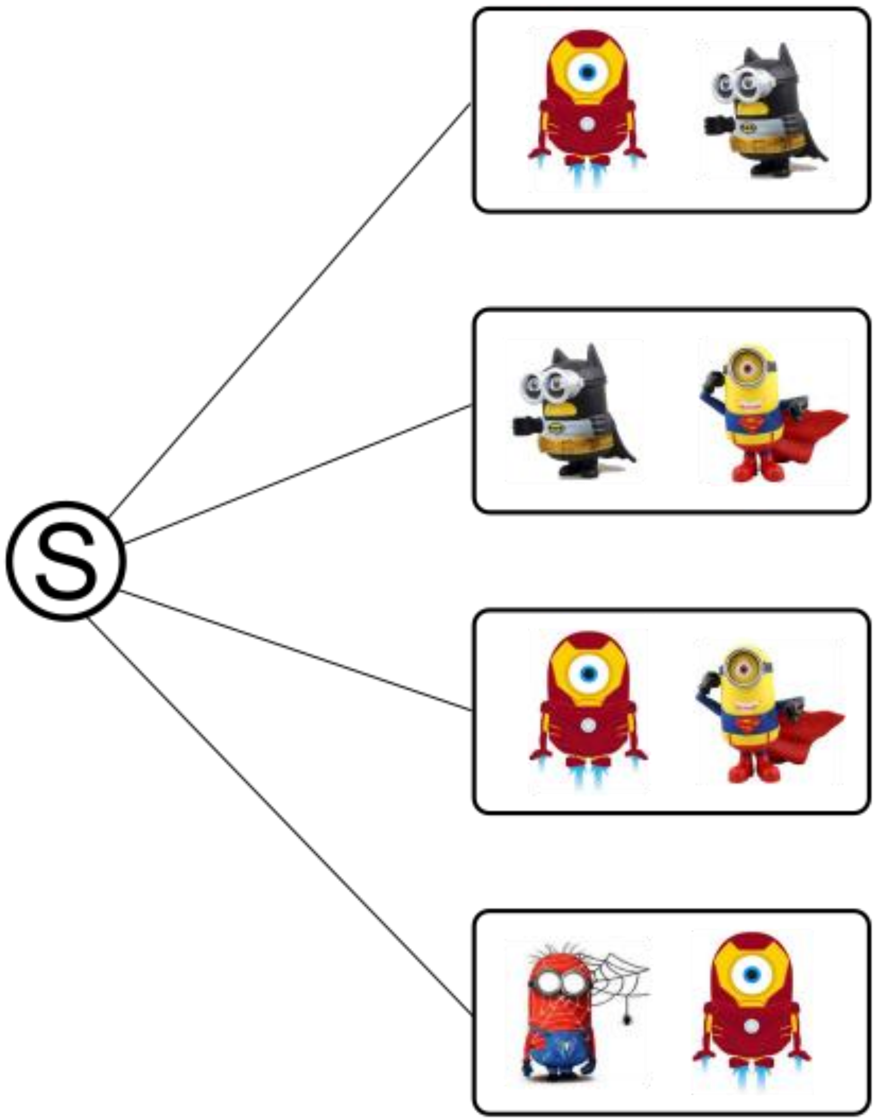
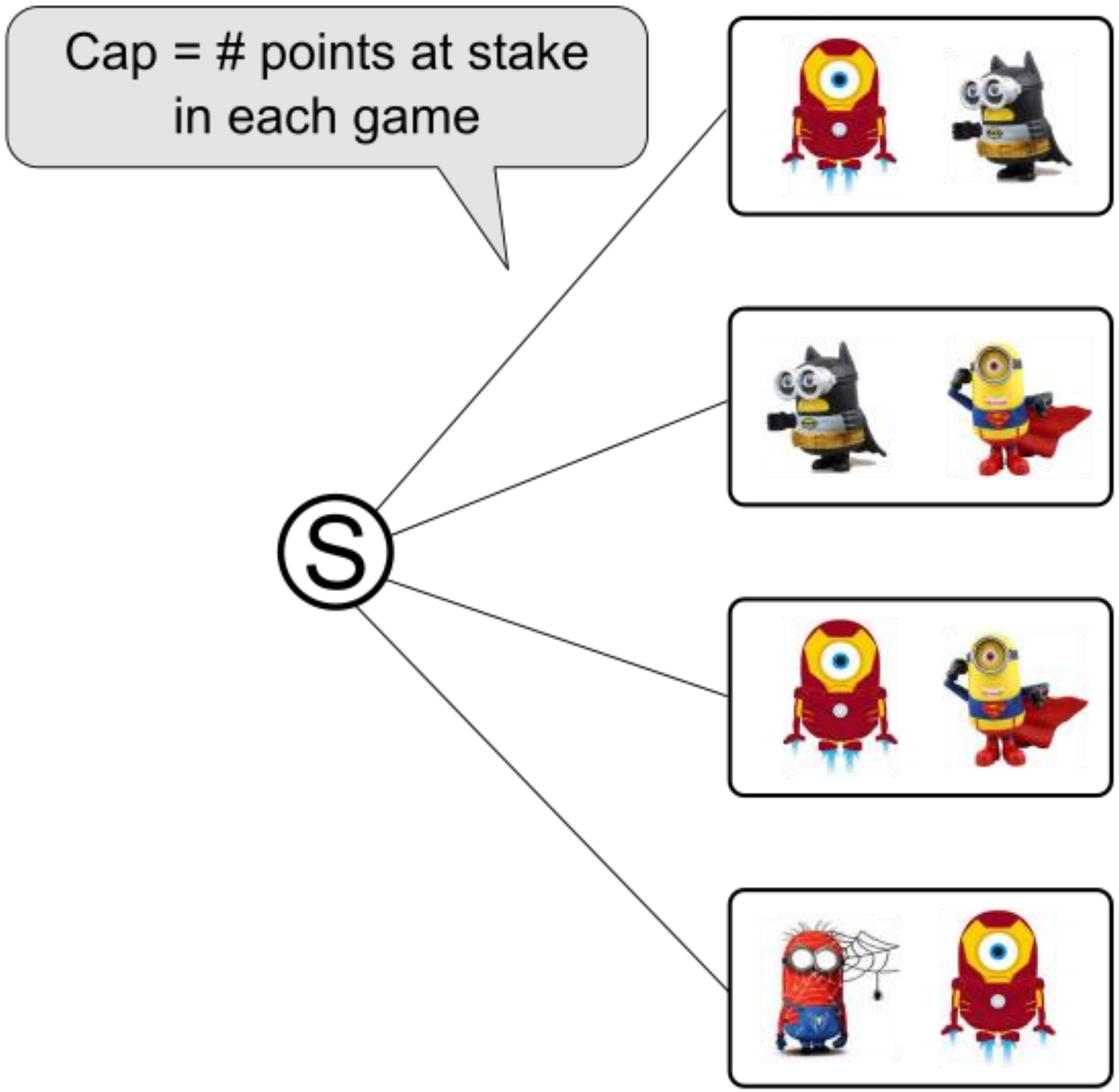
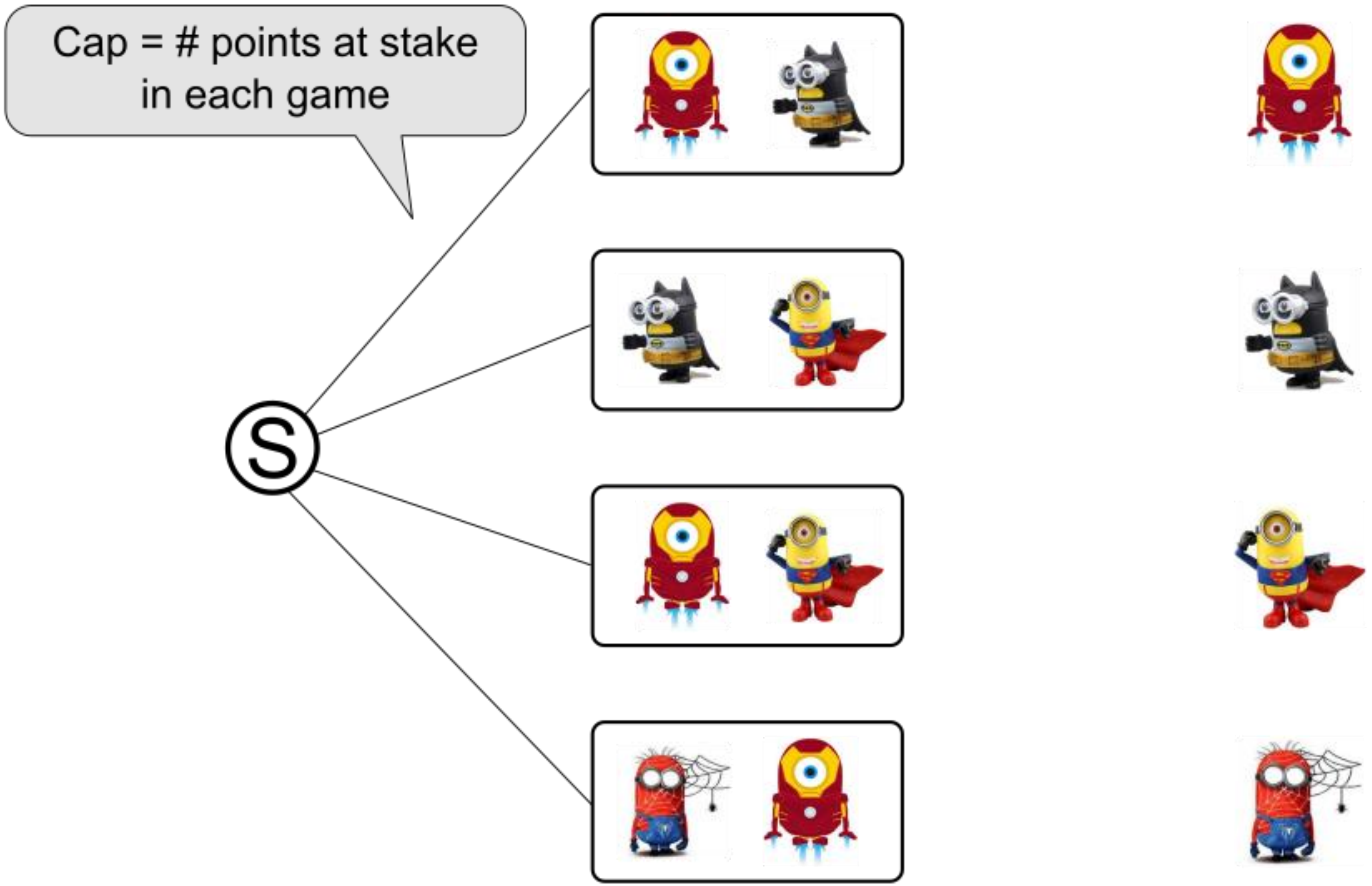
Doing so freezes the score of 

Step 2: Set up a flow network to check for a winning schedule.




S


Cap = # points at stake in each game
S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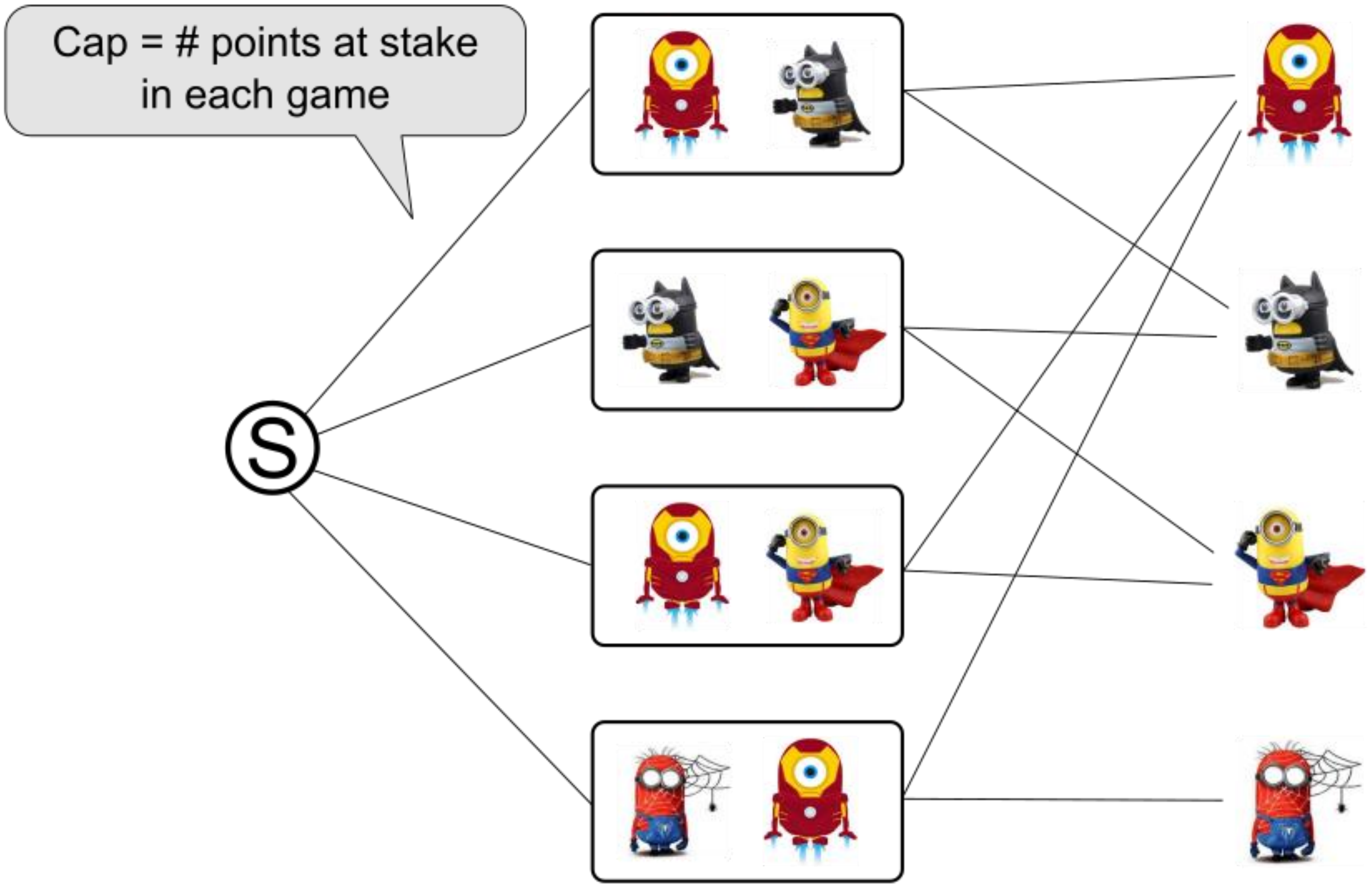
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in each game
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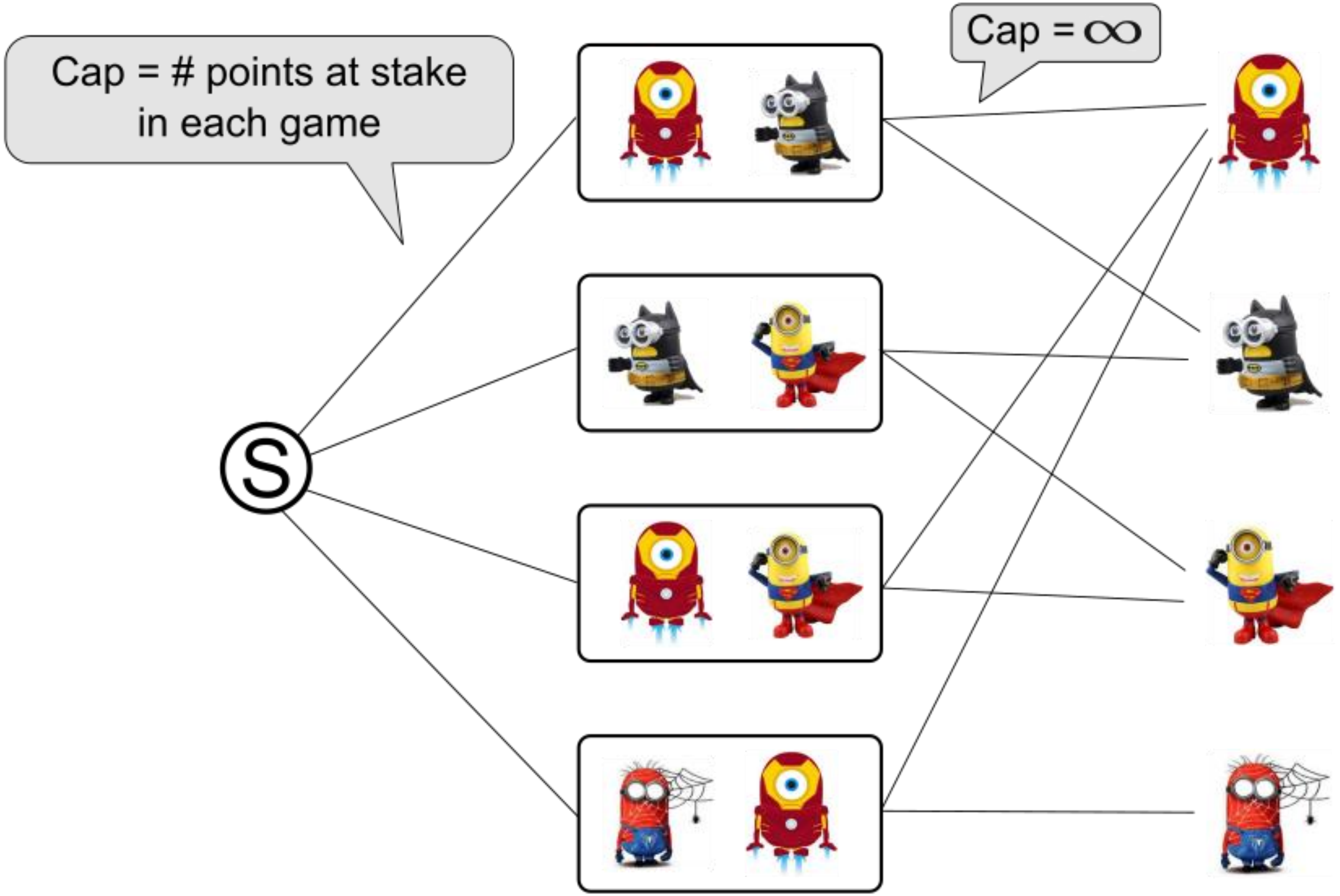
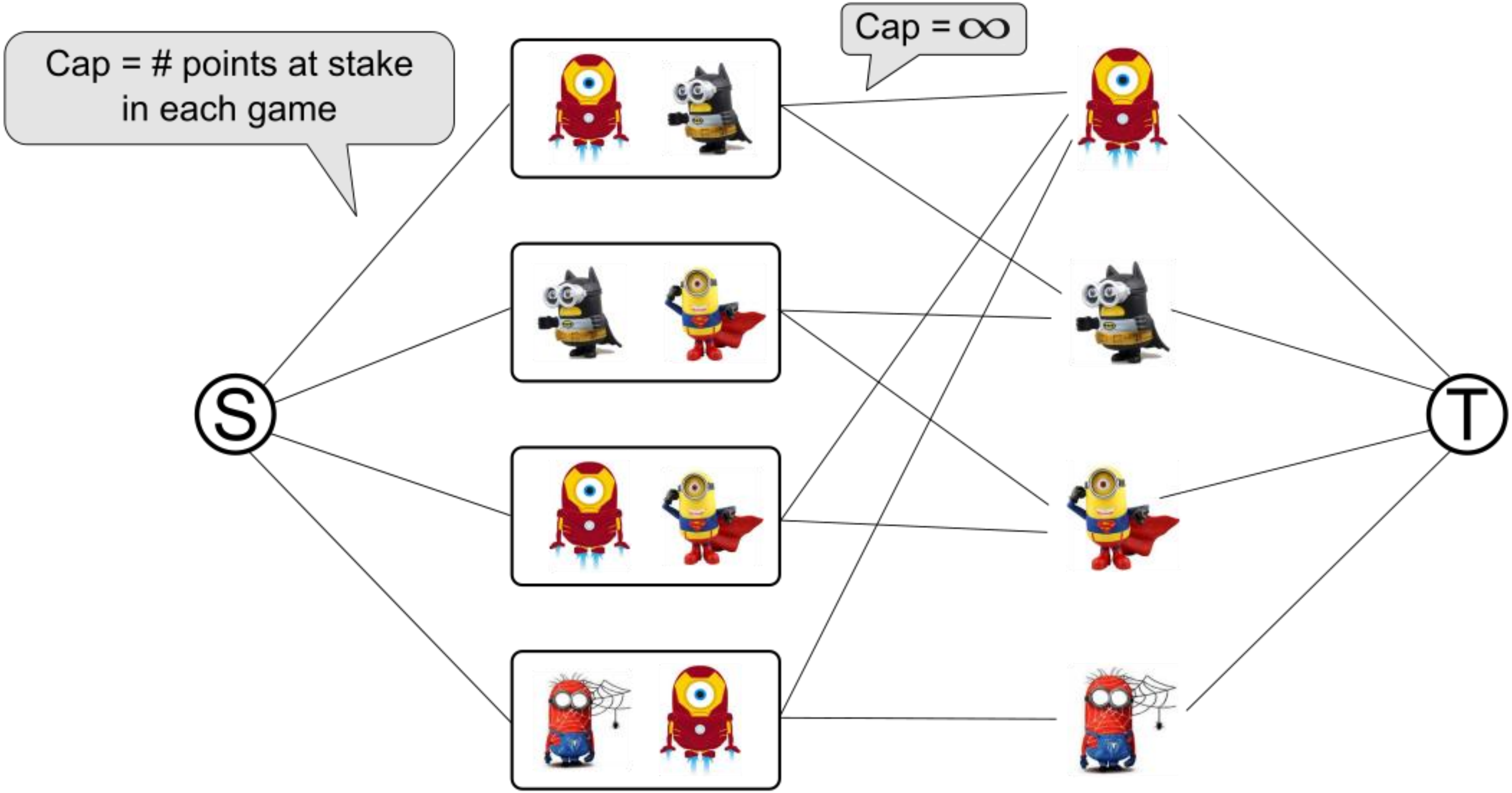
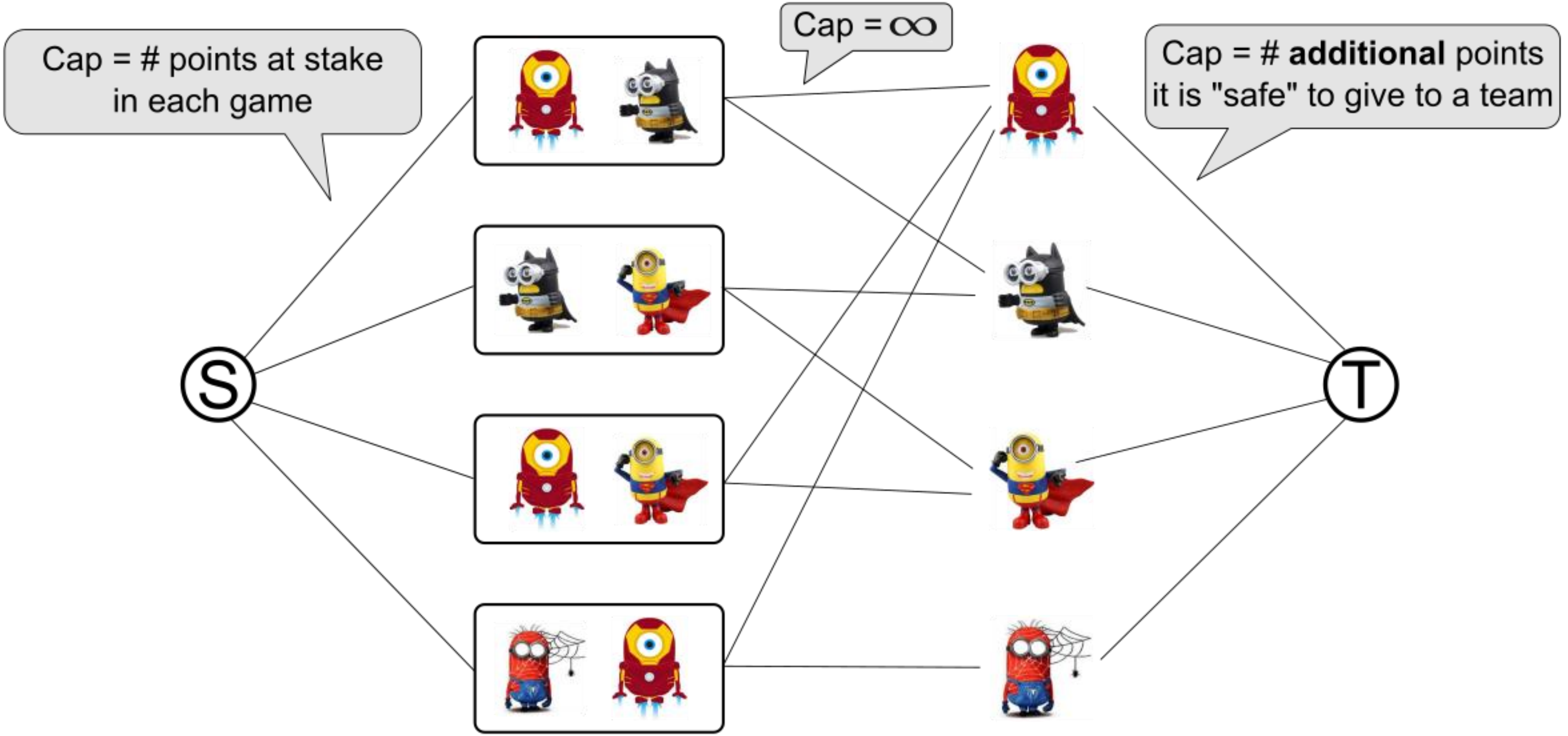
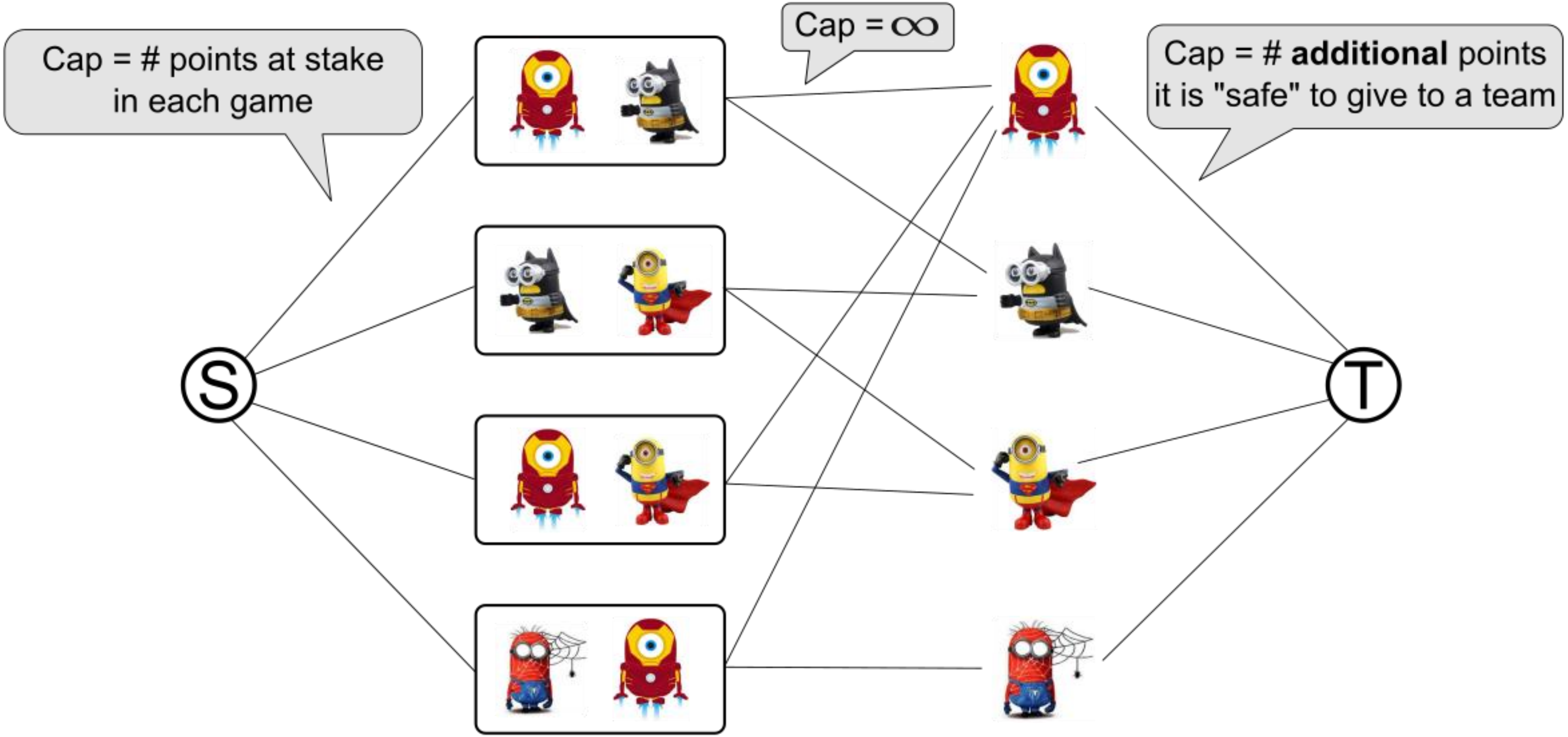

Cap = # points at stake
in each game
Cap = ∞
S


Cap = # points at stake in each game
Cap = ∞
S
T


Cap = # points at stake
in each game
Cap = ∞
Cap = # additional points
it is "safe" to give to a team
S
T



There is a max flow that saturates the edges of S ⇔ There is a winning schedule.

The minion championship used a $\{(0,1),(1,0)\}$ point system.

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What about other point systems?

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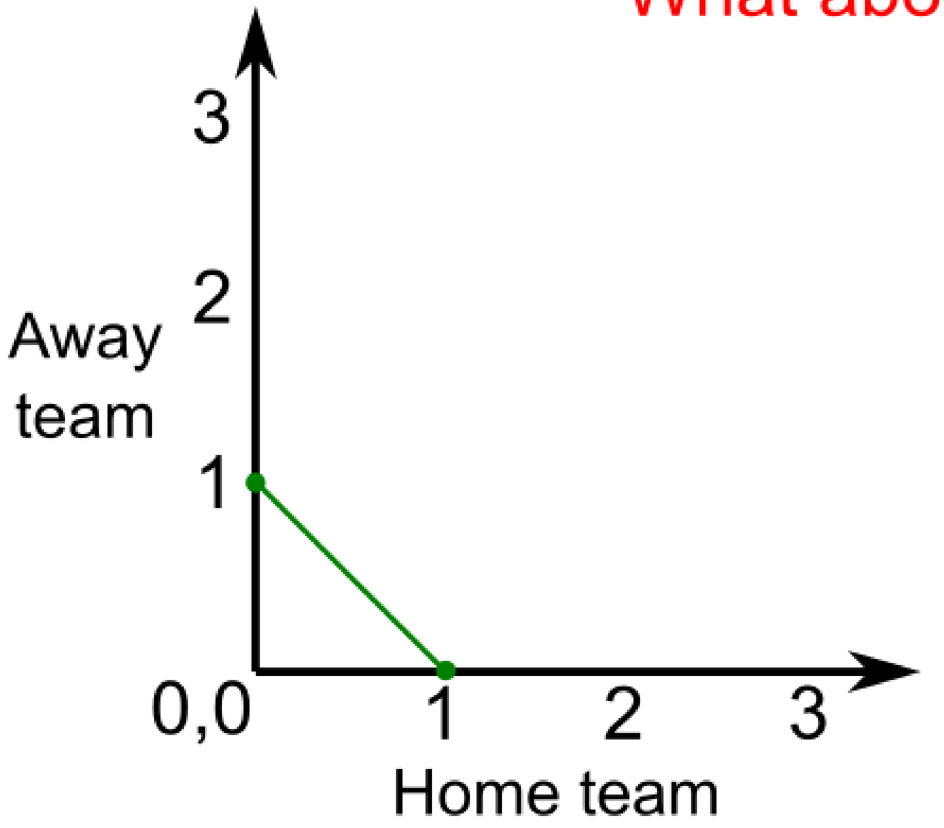

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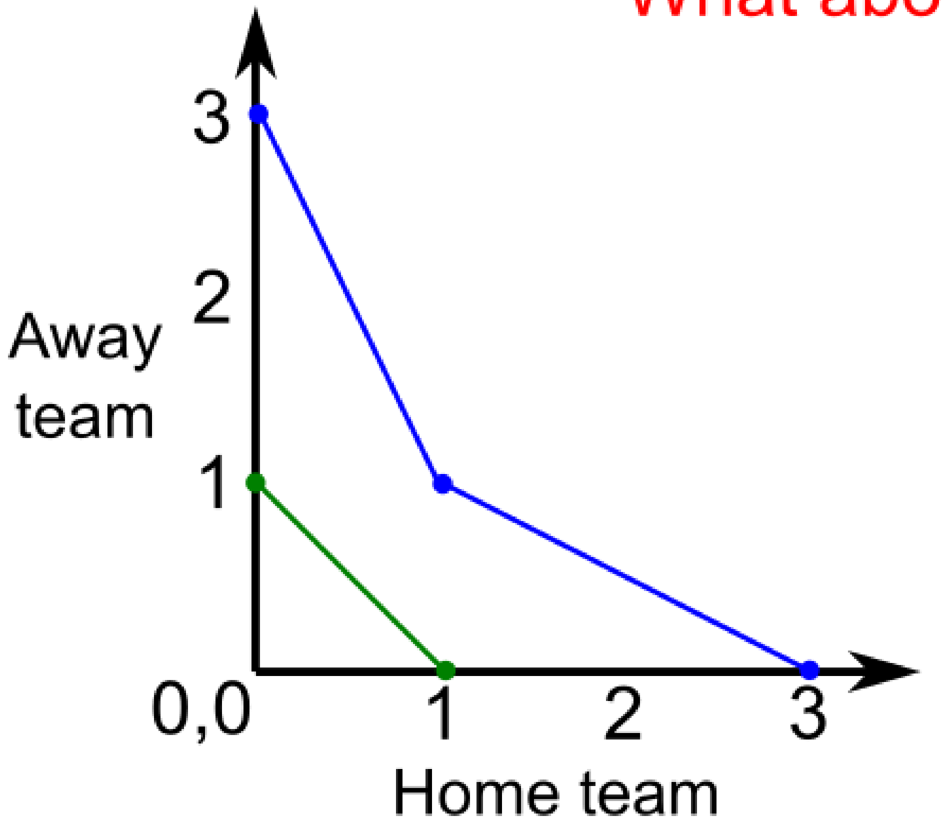

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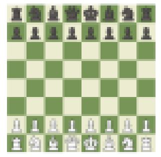


{(0,2),(1,1),(2,0)}

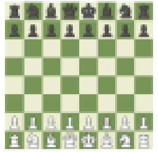
The minion championship used a {(0,1),(1,0)} point system.

What about other point systems?

{(0,1),(1,0)}

{(0,3),(1,1),(3,0)}

INDIAN PREMIER LEAGUE



{(0,2),(1,1),(2,0)}

{(0,3),(1,2),(2,1),(3,0)}

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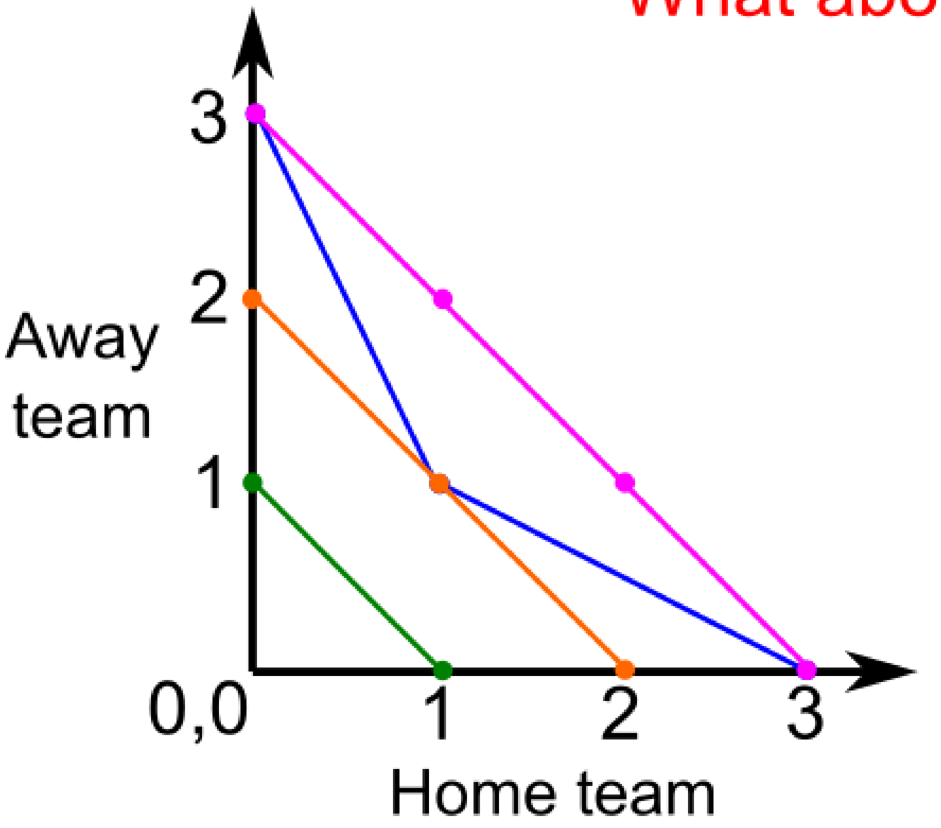
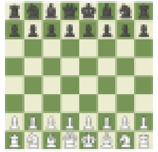


{(0,1),(1,0)}

{(0,3),(1,1),(3,0)}



{(0,2),(1,1),(2,0)}

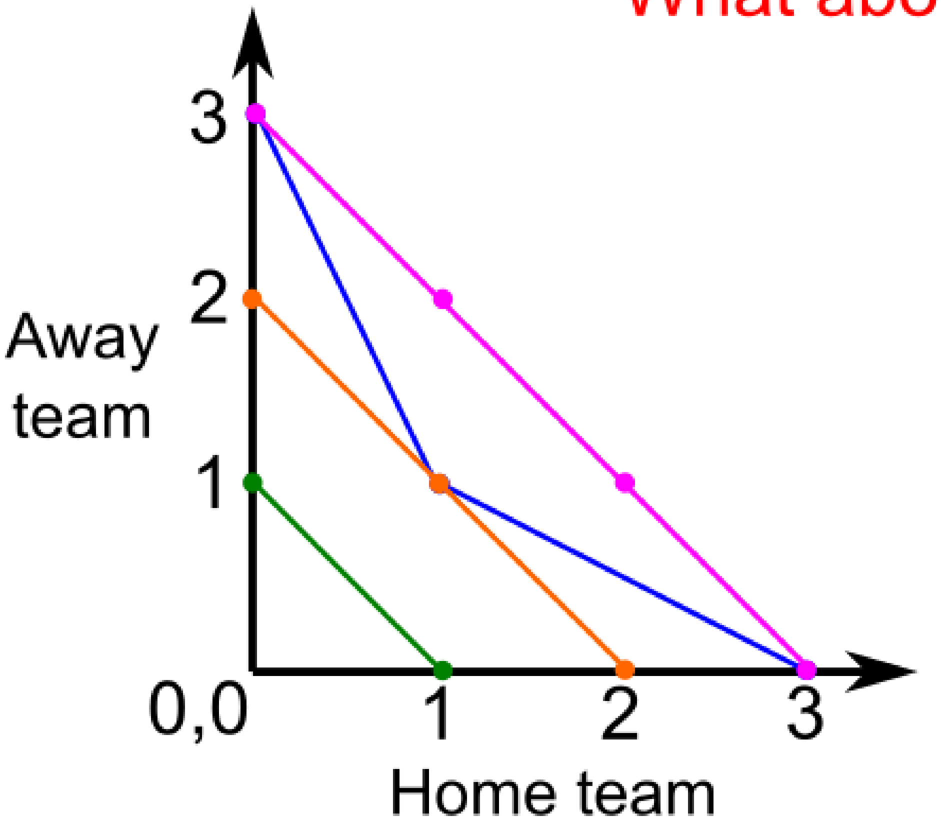
{(0,3),(1,2),(2,1),(3,0)}

[Kern and Paulusma, Disc. Opt. 2004]

Elimination problem is NP-complete for all point systems except for those that "line up nicely".

The minion championship used a {(0,1),(1,0)} point system.

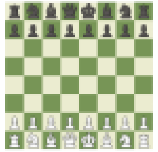
What about other point systems?



{(0,1),(1,0)}

{(0,3),(1,1),(3,0)}

INDIAN PREMIER LEAGUE



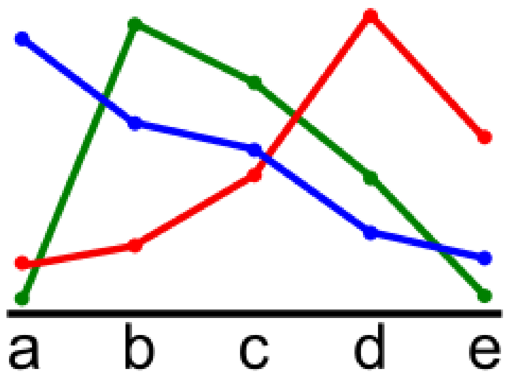
{(0,2),(1,1),(2,0)}

{(0,3),(1,2),(2,1),(3,0)}

Football is computationally harder than chess and ice hockey.

Next Time

Circumventing negative results with structured preferences



References

- “Sports elimination via max flow” with IPL teams: https://www.youtube.com/watch?v=XK6qZjHWo9A

- When it’s easy to recognize the *existence* of a beneficial manipulation but hard to *find* a manipulative vote.

  “Search versus Decision for Election Manipulation Problems”
  Hemaspaandra, Hemaspaandra, and Menton
  https://dl.acm.org/doi/10.1145/3369937